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Research Article

An Age Structure Model With Age-Migration Flow (A Mathematical Population Model)

Pawan Kumar Malviya[1 *] , Dr. Archana Jadon[2]

1, * 2Department of Mathematics IES University, Bhopal (M.P.)
Email: 1malviyapawan734@gmail.com

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ARTICLE INFO	ABSTRACT

This study deals with an age structured model with migration effect to study the dynamics of human population. The proposed model is a modified form of Mckendrick Von-Foerster equation (M-F Model) which has a partial differential equation of first order with initial condition and boundary condition. The initial condition is defined by the number of people of zero age and boundary condition given by the population at initial time, by using the partial differential equation defined as the dynamical flow that affect the dynamics of population density per ages, that means migration between ages, deaths per ages and total migration in the system. Our model is a deterministic model, in which we ignored the fertility rate and births are determined by birth rate. The variables in this model are continuous variables with range R^+ □{0} except for age migration whose range is R . A continuous time model is taken into account for the solution of the age structured model with some condition. The modified model allows forecasting the age dependent population density in the social system and beneficial in resources allocation and government policies.

Keywords: Population Growth, mathematical description, An Age Structure Model Arithmetic Increase

1. Introduction

The simplified population models usually start with four demographic processes including death, birth, immigration and emigration. These models are age unstructured and therefore ignore the attributes of individuals and especially the age pattern and composition of population numbers and consider only the number of people in the population and the members of population considered equally to die or reproduce For the study of Human Population Dynamics, age structured models are very important that cannot be ignored. There are various biological examples where the age play an important role in human population dynamics and it was McKendrick and von Foerster. They independently proposed a partial differential equation which plays a unifying role in population dynamics depending on age. The M-F equation is derived under the condition that age and time are continuous variable and the population is closed to migration, the spatial variation of population theory time and stochastic effect are not considered. In this equation it is usually assumed that only female is counted and describes the dynamics of such one sex population of age density. Males are present for reproductive purposes but are not specifically taken into consideration. In this approach the females of population have a biologically well defined beginning and the end to their reproductive careers, while the reproductive behavior of males is difficult to quantify.

2. McKendrick-Von Foerster Model

The M-F equation describes the dynamics of female population (one sex) using age structure or age distribution of function N(t,a) . The quantity N(t,a) da defines the number of individuals at time t with age between a and a □□da , and represent some kind of smoothing or statistical average of the true integer-valued population size. More precisely,

$$N(t,a) = \frac{number\ of\ individuals\ aged\ a\ to\ a + da\ at\ time\ t}{da}$$

In particular the number of individuals in the age bracket 1 2 (a ,a) at time t is given by

$$\int_{a_1}^{a_2} N(t,a)\,da \qquad (1)$$

So that the total population at time t is

$$P(t) = \int_0^{\infty} N(t,a)\,da \qquad (2)$$

To obtain this equation M-F equation, assumed that a small time increment dt has passed such that the age of each individual in the population is increased by da (where the obvious requirement that clock time t and age time a be measured in the same units is imposed). Hence the age distribution N(t □□dt,a) at time t □□dt would be same as it was at time t if everyone was da younger:

$$N(t+dt,a) = N(t,a-da) \qquad (3)$$

However during the time interval dt there would be a loss of individuals in each age-group due to death. This is given by□(t,a)N(t,a)da, where □(t,a) is the prescribed age specific death-rate (mortality function), that is, the death rate at age a and time t per unit population of age a . Thus equation (6.3) becomes

$$N(t+dt,a) = N(t,a-da) - \mu(t,a)\,N(t,a)\,da \qquad (4)$$

Assuming N is differential everywhere and expanding the two expression of N about t and a yields

$$N(t,a) + \frac{\partial N}{\partial t}dt + \ldots = N(t,a) + \ldots - \mu(t,a)\,N(t,a)\,da \quad (5)$$

While discarding higher power in da and dt and noting that da □□dt leads to MF equation

$$\frac{\partial N(t,a)}{\partial a} + \frac{\partial N(t,a)}{\partial t} + \mu(t,a)\,N(t,a) = 0 \qquad (6)$$

Obviously the solution to this hyperbolic partial differential equation can only be determined by specifying two auxiliary conditions:

An initial condition on the age distribution at some specified time, usually taken to be the age distribution at t □□o:

$$N(0,a) = \phi(a) \qquad (7)$$

and the boundary condition N(t,o) , denoting the number of births at any time t. The change occurring in the population of age a at time t, over a time interval of length h, is proportional to the size of the population and the length of the interval. Thus

$$N(t+h,a+h) - N(t,a) = -\mu(t,a)\,N(t,a)h \qquad (8)$$

Again assuming N is differentiable, Taylor expanding the led term and passing to the limit h□o yields the M-F equation (6)
The number of individual introduced into the population in the time interval (t, t □□h) is given by

$$\square\, h\int_0^{\infty} B(t,a)\,N(t,a)\,da \qquad (9)$$

where hB(t,a) is the average number of births produced by a female of age a in the interval (t, t □□h) with B(t,a) being the prescribed age specific fertility (also called fertility or fecundity function) of the population. By the definition, B(t,a) is the average number of off spring produced per unit time, by an individual of age a at time t.

If the birth rate b(t) is defined as the rate of addition of newborns (individual aged 0) to the population at time t, then clearly,

$$b(t) = N(t, 0) = \int_0^{\infty} B(t, a)\, N\,(t, a)\, da \tag{10}$$

As before, the initial condition that is N(0,a) □□(a) , is required for unique solution of equation (6.6) to be specified.
□(a) is most often chosen to be smooth function, becoming zero for large a , say a □□k , where k is a fixed constant representing the maximum life span of population. The assumption that there is a finite maximum attainable age (for example 100 years for human populations), ensures that the integral (2) and (11) are evaluated over finite intervals, since necessarily N(t,a) □□0 for a □□k .
Equations (2), (6), (8) and (11) constitute the McKendrick-Von Foerster (M-F) model of age-dependent or age-structured population growth and will be the focal point of this treatment of human population dynamics.

3. Continuous Time Non Age Structured Model

This section emphasis with a continuous time non age-structure model has a differential equation that permits performing predictions with migration through in human population dynamics. This model admits a relationship among the population density, births, deaths and migration. If P(t) is population at time t , B(t) is number of births at time t , D(t) is number of deaths at time t and g(t) is migration flow then the change of the population can be described as:

$$\left.\begin{aligned} \frac{dP(t)}{dt} &= B(t) - D(t) + g(t) \\ P(t) &= P_0 \end{aligned}\right\} \tag{11}$$

Where 0 P is the population at initial time 0 t . However, if b(t) and d(t) are crude birthrate and crude death rate respectively then B(t) and D(t) can be obtained as:

B(t) □□b(t) □P(t) and D(t) □□d(t) □P(t) (12)

Now we analyses input variable, crude birthrate b(t) and crude death rate d(t) from the above function. The function may be got from the present data-bases through dividing the B(t) andD(t) by the corresponding population density P(t) , where g(t) is an input variable and also time-dependent at t, then equation (12) can be rewritten as:

$$\left.\begin{aligned} \frac{dP(t)}{dt} &= \left[\, b(t) - fd(t)\, \right]. P(t) + g(t) \\ P(t_0) &= P_0 \end{aligned}\right\} \tag{13}$$

Non age- Structured Model is a good application to calculate forecasting the population in the social systems. Equation (6.13) is a first order Linear Equation with initial condition and has an explicit solution for 0 t □□t is as given below

$$P(t) = \exp\left(\int_{t_0}^{t} \left(b(y) - d(y)\, dy\right) \right) \left(P_0 + \int_{t_0}^{t} \exp\left(- \int_{t_0}^{y} \left(b\,(z) - d\,(z)\, dz\right) \right) g(y)\, dy \right) \tag{14}$$

4. Formulation of Modified Mckendrick-Vonfoerster Model

Let N(t,a) be the population density with respect age a of a population at time t . The population between the ages a □da, (da □□0)and a is given by $\int_{a-da}^{a} N(t, y)\, dy$, and taking N(t,a) is the smooth function of (t,a) .

Then the total population P(t) in the system is given by $P(t)=\int_{a-da}^{a} N(t,\,y)\,da$ □. And population N(t,a) will be zero for the high values of age a i.e.

$$\lim_{a\to\infty} N(t,\,a)=0 \;\forall\; t\ge t_0$$

Let n1(t,a) denotes the population between the ages a □da and a , and 2 n (t,a) is the population between the ages a and a □□da per time and age unit. Now to compute the 1n (t,a) and 2 n (t,a) , using the hypothesis applicable in . Consequently, 1n (t,a)will be directly proportional to the population at time t □□dt and at the age, a □□da and inversely proportional to finite increment da . If c is the proportionality constant then

$$n_1(t,\,a)=c\,\frac{N(t-dt,\,a-da)}{da} \qquad (15)$$

$$n_2(t,\,a)=c\,\frac{N(t-dt,\,a)}{da} \qquad (16)$$

Now a finite difference Equation must be written like equation (12) must be written for N(t,a) by taking the corresponding limits, zero for time increments dt and age increments da .Then by using the finite difference scheme we find a difference equation, that is

$$N(t,\,a)=N(t-dt,\,a)+\big(\,(n_1(t,\,a)-n_2(t,\,a)-\eta(t,\,a)+m(t,\,a\,)\,\big).\,dt \qquad (17)$$

The population density is a balance from (6.17) for each age a . It is clearthat birth flow is not show in (17), but plays an important application with boundary condition for zero age population. It means that the corresponding hypothesis how to obtain the input and output variables in (17). Where m(t,a) represent age migration flow and □(t,a) denotes the no. of deaths per unit of time and age. The way to calculate □(t,a) is similar to continuous model:

$$\eta(t,\,a)=\mu(t,\,a).\,N(t-dt,\,a) \qquad (18)$$

Where □(t,a) is the age-death rate and $\lim_{a\to\infty}\mu(t,\,a)=\infty\;\forall\;t\ge t_0$

If (15), (16) and (18) put in (17), we get the finite difference equation. But to describe the continuous equation, applying Taylor approximation in equation (15), we get

$$n_1(t,\,a)=\frac{c}{da}\big(N(t-dt,\,a-da)\,\big)\approx\frac{c}{da}\left(N(t-dt,\,a)-\frac{\partial N(t-dt,\,a)}{\partial a}\,da\right) \qquad (19)$$

Again, substituting equations (16), (18) and (6.19) in equation (17), doing corresponding hypothesis, we get

$$N(t,\,a)=N(t-dt,\,a)+\left[\frac{c}{da}\left(N(t-dt,\,a)-\frac{\partial N(t-dt,\,a)}{\partial a}\right)da \right.$$
$$\left.-\frac{c}{da}\,N(t-dt,\,a)-\mu(t,\,a)\,N(t-dt,\,a)+m(t,\,a)\right]dt$$

$$\frac{N(t,\,a)-N(t-dt,\,a)}{dt}+c\,\frac{\partial N(t-dt,\,a)}{\partial a}=-\mu(t,\,a)\,N(t-dt,\,a)+m(t,\,a) \qquad (20)$$

Taking limit dt□0 in equation (19), we have

$$\lim_{dt \to 0} \frac{N(t,a) - N(t-dt,a)}{dt} + c \lim_{dt \to 0} \frac{\partial N(t-dt,a)}{\partial a} = \lim_{dt \to 0} \left[\mu(t,a)\, N(t-dt,a) + m(t,a)\right]$$

$$\frac{\partial N(t,a)}{\partial t} + c\,\frac{\partial N(t,a)}{\partial a}\,\mu(t,a)\, N(t-a) + m(t,a) \tag{21}$$

The corresponding hypothesis to used as in continuous model, is to include the births, B(t) that take place at a □□o then

$$N(t,0) = B(t) = b(t).\,P(t)\,\square$$

$$N(t,0) = b(t)\int_0^\infty N(t,a)\,da \tag{22}$$

Let u(a) is a function that provide the population density of aged a at initial time o t then

$$N(t_0, a) = u(a) \tag{23}$$

For the higher values of age variable, o N(t ,a) will be close to zero.If the above equations are contained in the age structure population model as the boundary condition for u(a) and combining the equations (21), (22) and (23) then we get an age structured population model which is a modification of classic M-F Model after age-migration flow

$$\left.\begin{array}{l} \dfrac{\partial N(t,a)}{\partial t} + c\,\dfrac{\partial N(t,a)}{\partial a} = -\,\mu(t,a)\, N(t,a) + m(t,a) \\ N(t,0) = b(t)\displaystyle\int_0^\infty N(t,a)\,da \\ N(t_0, a) = u(a) \end{array}\right] \tag{24}$$

5. A Solution to the Age-Structured Model

In this section we obtain a solution for the equation (24). For this observe that the equation (21) is a first order partial differential equation, then the characteristic differential Equation of equation (21) is given by

$$\frac{dt}{1} = \frac{da}{c} = \frac{dN}{-\,\mu(t,a)\, N(t,a) + m(t,a)} \qquad \square\square\square\square\square\square(25)$$

From equation (25), independently we have three ordinary differentials equations as:

$$\frac{da}{dt} = c\,\square \qquad \square\square\square\square\square\square\square \quad \square\square\square\square\square(26)$$

$$\frac{dN}{da} = \frac{1}{c}\left(-\,\mu(t,a)\, N(t,a) + m(t,a)\right) \qquad \square\square\square\square\square\square\square(27)$$

$$\frac{dN}{dt} = --\,\mu(t,a)\, N(t,a) + m(t,a)\square \qquad \square\square\square\square\square\square\square(28)$$

Now integration of the equation (26) gives,

$$a - ct = k_1 \Rightarrow t = \frac{a-k_1}{c} \; where \; k_1 \text{ is constant}$$

Again using this result in equation (27) we have

$$\frac{dN}{da} = -\frac{1}{c}\mu\left(\frac{a-k_1}{c}, a\right) N(t,a) + \frac{1}{c} m\left(\frac{a-k_1}{c}, a\right) \tag{29}$$

Equation (29) is a first order linear differential equation and has a known solution

$$N(t,a)=\exp\left(-\frac{1}{c}\int\mu\left(\frac{a-k_1}{c},a\right).da\right)$$
$$\times\left(\frac{1}{c}\int\mu\left(\frac{a-k_1}{c},a\right).\exp\left(\frac{1}{c}\int\mu\left(\frac{a-k_1}{c},a\right).da\right)da+k_2\right) \qquad (30)$$

Where 2 k is another constant and can be written as 2 1 k □□f (k) , where function 1 f (k) depends on boundary and initial condition then formal solution

$$N(t,a)=\exp\left(-\frac{1}{c}\int\mu\left(\frac{a-k_1}{c},a\right).da\right)$$
$$\times\left(\frac{1}{c}\int m\left(\frac{a-k_1}{c},a\right).\exp\left(\frac{1}{c}\int\mu\left(\frac{a-k_1}{c},a\right).da\right)da+f\,(a-ct)\right) \qquad (31)$$

Now defining the following auxiliary function:

$$R\,(a,a-ct)=\frac{1}{c}\left[\int\mu\left(\frac{y-(a-ct)}{c}\;\;y\right)dy\right]_{y=a} \qquad (32)$$

$$S\,(a,a-ct)=\frac{1}{c}\left[\int\mu\left(\frac{y-(a-ct)}{c},y\right).\exp\left(\frac{1}{c}\int\mu\left(\frac{y-(a-ct)}{c},y\right)dy\right)dy\right]_{y=a} \qquad (33)$$

Using equation (32) and (33) in (31), then we have

$$N(t,a)=\exp\left(-R\,(a,a-ct)\right).\;\left(S\,(a,a-ct)+f\,(a-ct)\right) \qquad (34)$$

Now, to determine the function f taking boundary condition (22) in account. Thus

$$N(t,0)=\exp\left(-R\,(0-ct)\right).\;\left(S\,(0-ct)+f\,(-ct)\right)$$

And

$$b(t)\int_0^\infty N(t,a)\,da=b(t)\int_0^\infty\exp\left(-R\,(a,a-ct)\right).\left(S\,(a,a-ct)+f\,(a-ct)\right)da$$

$$\exp\left(-R\,(0-ct)\right).\;\left(S\,(0-ct)+f\,(-ct)\right)$$

$$=b(t)\int_0^\infty\exp\left(-R\,(a,a-ct)\right).\left(s\,(a,a-ct)+F(a-ct)\right)da \qquad (35)$$

To find the solution to the model, using an approach that if boundary condition considered as a part of the final solution and total population is given by

$$P(t)=\int_0^\infty N(t,a)\,da$$

Then using equation (14) in equation (22), we have

$$\exp\left(-R\,(0,-ct)\right).\left(S(0,-ct)+f(-ct)\right)=b(t).\;p(t) \qquad (36)$$

Observe that equation (6.36) is valid only for t □□to . Taking 0 z □□□ct□t □□□(z / c) □□t
Then

$$\exp\left(-R\,(0,z)\right).\left(S(0,z)+f(z)\right)=b\left(-\frac{z}{c}\right).\,P\left(-\frac{z}{c}\right)$$

$$f(z)=b\left(-\frac{z}{c}\right).\,P\left(-\frac{z}{c}\right).\exp\left(R\,(0,z)\right)-S(0,z) \qquad (37)$$

Conveniently if we taking a new change z □□a □ct thus $a-ct=-ct \Rightarrow -\frac{(a-ct)}{c}=t-\frac{a}{c}\geq t_0$ and equation (37) becomes

$$f(a-ct)=b\left(t-\frac{a}{c}\right).P\left(t-\frac{a}{c}\right).\exp\left(R(0,a-ct)\right)-S(0,a-ct) \qquad (38)$$

And if, moreover

$$S(a,a-ct)-S(0,-ct)=\frac{1}{c}\int_0^a \mu\left(\frac{y-(a-ct)}{c},y\right)dy \qquad (39)$$

Using equation (38) in equation (34), we have

$$N(t,a)=\exp\left(-R(a,a-ct)\right).\left((S(a,a-ct)-(S(0,a-ct))\right. \\ \left.+b\left(t-\frac{a}{c}\right).P\left(t-\frac{a}{c}\right).\exp\left(R(0,a-ct)-R(a,a-ct)\right)\right) \qquad (40)$$

$$P\left(t-\frac{a}{c}\right)=\exp\left(\int_{t_0}^{t-a/c}(b(y)-d(y))\,dy\right).\left(P_0\int_{t_0}^{t-a/c}\exp\left(-\int_{t_0}^{t-a/c}(b(z)-d(z))dz\right)g(y)dy\right) \qquad (41)$$

If we take boundary condition into account then we can find a partial solution $N_1(t,a)$ which is valid when t-a/c□ t□□ that is a □□c(t □t₀) . For this Using equations (32), (33), (39) and (41) in Equation (40) and after some simplification we find 0

$$\begin{aligned} N_1(t,a)=&\,b\left(t-\frac{a}{c}\right).\exp\left(\int_{t_0}^{t-a/c}(b(y)-d(y))\,dy)-\frac{1}{c}\int_0^a\mu\left(\frac{y-(a-ct)}{c},y\right)dy\right) \\ &\times\left(P_0+\int_{t_0}^{t-a/c}\exp\left(-\int_{t_0}^{y}(b(z)-d(z))\,dz\right)g(y)dy\right) \\ &+\frac{1}{c}\int_c^a\left(m\frac{y-(a-ct)}{c},y\right).\exp\left(\frac{1}{c}\int_c^y\mu\left(\frac{z-(a-ct)}{c},z\right)dz\right)dy \end{aligned} \qquad (42)$$

Now taking initial condition equation (23) in equation (34), we get

$$N(t_0,a)=u(a)=\exp\left(-R(a-ct_0)\right).\left(S(a,a-ct_0)+f(a-ct_0)\right),\ a\geq 0 \qquad (43)$$

Conveniently, taking two consecutive change of variables $z=a-ct_0\ (thus,\ a=z+ct_0\geq 0)\Rightarrow a-ct+ct_0=a-c\,(t-t_0)\geq 0$ are performed, and then f (a □ct) is obtained as:

$$f(a-ct)=\exp\left(-R(a-c\,(t-t_0),a-ct)\right).u\left(a-c\,(t-t_0)\right)-S(a-c\,(t-t_0),a-ct) \qquad (44)$$

This result is valid when 0 a □□c(t □t) . If we put equation (44) in equation (34) then after some simplification we find a partial result which is valid for 0 a □□c(t □t₀) is given by

$$\begin{aligned} N_2(t,a)=&\exp\left(-\frac{1}{c}\int_{a-c(t-t_0)}^{a}\mu\left(\frac{y-(a-ct)}{c},y\right)dy\right).u\left(a-c\,(t-t_0)\right) \\ &+\frac{1}{c}\int_{a-c(t-t_0)}^{a}\left(m\left(\frac{y-(a-ct)}{c},y\right).\exp\left(\frac{1}{c}\int_a^y\mu\left(\frac{z-(a-ct)}{c},z\right)dz\right)\right)dy \end{aligned} \qquad (45)$$

Thus the solution of the age structured model Equation (24) is given by

$$N(t,a)=\begin{cases} N_1\left(t,a\right) \ when\ a\le\ c\left(t-t_0\right) \\ N_2\left(t,a\right) \ when\ a\le\ c\left(t-t_0\right) \end{cases} \qquad (46)$$

Where N_1(t,a) and N_2 (t,a) are given by equation (42) and (45) respectively. If a □□c(t □t_o) or u(o) □□b(t) □P_o , then equation (42) and (45) should be same. That is, for a □□o the initial condition will be equalize the number of newborn at initial time
t □□t_o .
Further, the formal structure of age and time dependant variables, which is age death rate □(t,a) and age migration flow, m(t,a) can be defined for the practice of equation (46). If the □(t,a) depends only on age and independent to time then it can be expressed as

□(t,a) □□(a) (47)

And the migration flow can be considered as

$$m\left(t,a\right)=m_i\left(t\right).\ f_i\left(a\right)-m_e\left(t\right).\ f_e\left(a\right) \qquad (48)$$

Where, m_1(t) and m_e(t) are the immigration and emigration flows respectively and f_1(a) and f_e(a) are, respectively age immigrant family composition and age emigrant family composition. Both f_1(a) and f_e(a) defined as the proportions per age unit and have a social meaning.

6. Conclusion

This study presents a modified Mc-Kendrick –Von-Foerster age structured model consisting of a first order quasi linear partial differential model, for the study of dynamical structure of human population. For the modification of classical M-F model we consider the migration flow which is the migration between ages. Population density, which is dependant of age and time, is the essential variables of this model. The boundary condition is defined by the population at initial time and initial condition is given by number of people of zero age. All the input variables in this model are age and time dependant that influence the dynamics of age structured population. This model is a deterministic non- control model in which number of births are determined by birth-rate and we ignored the fertility rate. So, modified model is an abstract form of the classical M-F model or can be applied on both male and female population and allows forecasting the age dependant population in the social system and beneficial in resources allocation and Govt. policies. The partial differential equation of our age structured model has been defined using dynamical variables (deaths per age, migration between ages) that affect the population density dynamics per ages, of the system. The main hypothesis defined that the population migration between the ages is directly proportional to the population density of the preceding age to population density of computed one and inversely proportional to the age interval of the same age. This model can be used for the social needs along time for each age, for instance, schooling needs or necessary resources to assist senior citizens.

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