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Investigating The Influence Of Driver Misconceptions On Road Behaviour Using A Novel Lattice Hydrodynamic Model

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ARTICLE INFO **ABSTRACT**

The misconception of drivers' behaviour on the road has a significant impact on traffic affecting other drivers and leading to traffic jams and accidents. Consequently, a new hydrodynamic model has been developing for traffic flow to investigate the effects of misconception of drivers' behaviour on the road. Through Density-sensitivity phase analysis it has been observed that the stable region of traffic flow expands as the misunderstanding of drivers decreases.

Moreover, through nonlinear analysis, a kink-antikink soliton solution of the mKdV equation associated with the misconception effect has been identified. Finally, numerical simulations were executed to analyse the evolution of density and flux. The results and through differed graphs indicate that, as the misconception of drivers behaviour diminishes traffic flow becomes increasingly stable.

1. Introduction

In recent years, there's is increase in traffic day by day. It causing more congestion, pollution, time wasted and accidents Because of this, many research projects have started, offering different mathematical modelling to help to solve these problems[1, 2, 3, 4, 5, 6, 7] additionally number of mathematical car-following models [8, 9, 10, 11, 12] given by Wang, Chen, Zhang etc. and continuum models have been proposed by Yu[13],Mohan[14], Gupta[15],Jiang[16] Besson[17] etc. continuum. Various approaches have been introduced to enhance comprehension and control of the complex dynamics of traffic congestion. Nagatani [18] originally derived the lattice hydrodynamics model. Subsequently, significant research efforts have been dedicated to the examination of the Lattice Hydrodynamic (LH)model considering various factors such as probability of optimal current on traffic congestion [19], backward looking effect, memory effect [20, 21] delayed feedback control [22]. As literature available till now there is no research done on misconception of traffic behaviour. This research is needed to reduce congestion on roads, ensure safety on the roads, facilitate efficient flow of traffic, prevent accidents, and protect the lives of drivers, passengers and pedestrians. By providing clear guidelines for driving conduct, such as speed limits, lane discipline, and right-of-way protocols, traffic rules help to minimize confusion for smooth flow of traffic on the roads. In this paper, a novel approach has been introduced to explore the influence of driver misconceptions on road behaviour. Proposed method utilizes a lattice hydrodynamic model as a powerful tool in traffic flow analysis to simulate various scenarios where drivers possess different levels of misconception. By integrating principles from lattice hydrodynamics and cognitive science is aim to quantify how these misconceptions affect traffic flow congestion patterns and overall road efficiency.

Firstly, it sheds light on the role of driver cognition in shaping traffic dynamics highlighting the importance of accurate driver knowledge. Secondly, it provides insights into the potential benefits of addressing misconceptions through targeted educational initiatives and improved signage systems. Finally, by advancing our understanding of complex traffic phenomena this study contributes to the development of more resilient and adaptive transportation systems. Through proposed investigation our aim understands of how driver misconceptions impact road behaviour and ultimately contribute to the enhancement of traffic management strategies and road safety measures.

2. Proposed Model

Initially Nagatani [18] developed one-lane lattice hydrodynamic model, which is

$$\rho_j(t+\tau)-\rho_j+\tau\rho_0\left[\rho_j(t)v_j(t)-\rho_{j-1}(t)v_{j-1}(t)\right]=0 \tag{1}$$

$$\rho_j(t+\tau)v_j(t+\tau)=\rho_0 V(\rho_{j+1}) \tag{2}$$

After that, Zang[23],Huang[24], Peng[25],Zhou[26] ,Wang[27] and others also deigned new models with different factors in one-lane and two lanes. The stabilization effect of the density difference in the modified lattice [28] hydrodynamic model of traffic flow.

Here, Eqs. (1) and (2) are the dynamical equations for "the conservation of mass" and "flow evolution equation". In the context of time t, $\rho_j(t)$ stands for the local density and $v_j(t)$ corresponds to the velocity at the j^{th} location on one-dimensional lattice. The average density is ρ_0 and the driver's receptiveness is quantified by $a=1/\tau$. In the above expression, the "optimal velocity function" is

$$V(\rho_j(t))=\frac{V_{\max}}{2}[\tanh(\frac{1}{\rho_j}-\frac{1}{\rho_c})+\tanh(\frac{1}{\rho_c})] \tag{3}$$

where $V_{\max}$ and ρ_c denote the maximal velocity and the safety-critical density, respectively. In really of traffic, drivers often harbour various misconceptions that can impact their behaviour and decision-making on the road. Some common misconceptions include:

(a) Time-saving Illusions: Many drivers believe that aggressive makeover such as weaving through traffic or tailgating will save them time. However, studies have shown that such behaviours often result in minimal time savings while increasing the risk of accidents.
(b) Lane Choice Fallacies: Some drivers falsely believe that constantly switching lanes will help them progress faster. In reality, frequent lane changes can disrupt traffic flow and contribute to congestion.
(c) Overestimation of Abilities: Certain drivers overestimate their driving skills and underestimate the risks associated with distracted or reckless driving. This can lead to dangerous behaviour such as texting while driving or excessive speeding.
(d) Misunderstanding Right of Way: Misconceptions about right of way can lead to conflicts and confusion at intersections. Drivers may wrongly assume they have priority, leading to potentially hazardous situations.
(e) Ignorance of Traffic Laws: Lack of understanding or adherence to traffic laws can create unsafe conditions on the road. Drivers may be unaware of rules regarding signalling, yielding or speed limits, further contributing to congestion and accidents.
(f) Failure to Anticipate Traffic Patterns: Some drivers fail to anticipate traffic patterns and instead react impulsively to changing conditions. This can exacerbate congestion and increase the likelihood of collisions.

Addressing these misconceptions requires education, awareness campaigns, and enforcement of traffic laws. By promoting responsible driving behaviours for road safety can be improved and traffic flow can be optimized for everyone's benefit. Therefore proposed model is

$$\partial_t(q_j(t)) = a[\rho_0\alpha V(\rho_{j+1}(t)-\rho_j v_j(t)] + \frac{\lambda a}{\rho_0}[\rho_j(t)-\alpha\rho_{j+1}(t)] \tag{4}$$

Continuity equation remains same as that Eq. (1) After removing velocity $v_j(t)$ from Eq. (1) and (4), density equation obtained as

$$\partial_t^2(\rho_j(t))=a\rho_0^2\alpha[V(\rho_{j+1}(t)-\rho_j v_j(t)] + a\partial_t(\rho_j(t))+a\lambda[(1+\alpha)\rho_j-\rho_{j-1}(t)-\alpha\rho_{j+1}(t)] \tag{5}$$

λ is feedback coefficient as above and α is misconception coefficient. When λ=0 then it become Nagatani one- lane model.

3. Linear analysis

To investigate the linear-stability of the misconception effect of driver in traffic. The traffic-density and optimal-velocity under uniform conditions are taken as ρ_0 and $V(\rho_0)$ respectively. Let the steady state solution for homogeneous traffic flow's is following

$$\rho_j(t) = \rho_0 \tag{7}$$

$$v_j(t) = v(\rho_0) \tag{8}$$

$$\rho_j(t) = \rho_0 + y_j(t) \tag{9}$$

$$V(\rho_j(t)) = V(\rho_0) + V^{'}(\rho_0) y_j(t) \tag{10}$$

Let $y_j(t)$ be a small perturbation to the steady-state density on site-j.

Substituting $\rho_j(t) = \rho_0 + y_j(t)$ and $y_j(t) = e^{ikj+zt}$ in into Eq. (6) then get Eq(11) and Eq(12)

$$\begin{aligned}(\rho_j(t+2\tau) - 2\rho_j(t+\tau) + \rho_j(t) + a\tau^2\rho_0^2\alpha[V(\rho_{j+1}(t) - V(\rho_j(t)] + a\tau[(\rho_j(t+\tau) - \rho_j(t)]\\ -\lambda\tau^2[(1+\alpha)\rho_j - \rho_{j-1}(t) - \alpha\rho_{j+1}(t)]\end{aligned} \tag{11}$$

$$z^2 + a\alpha\rho_0^2 V^{'}(\rho_0)[e^{ik} - 1] + az + a\lambda[(1+\alpha) - e^{-ik} - \alpha e^{ik}] \tag{12}$$

After substituting $z = z_1(ik) + z_2(ik)^2 ...$ into Eq. (12), got that On comparing the coefficients of (ik) and $(ik)^2$ obtained that

$$z_1 = -\alpha\rho_0^2 V^{'}(\rho_0) + \lambda[(1-\alpha) \tag{13}$$

$$z_2 = 3z_1^2 - \frac{a\alpha\rho_0^2 V^{'}(\rho_0)}{2} - \frac{\lambda[(1+\alpha)}{2} \tag{14}$$

$$a = \frac{3(\alpha\rho_0^2 V^{'}(\rho_0)) + \lambda(1-\alpha))^2}{\alpha\rho_0^2 V^{'}(\rho_0) + \lambda(1+\alpha)} \tag{15}$$

$$\tau = \frac{\alpha\rho_0^2 V^{'}(\rho_0) + \lambda(1+\alpha)}{3(\alpha\rho_0^2 V^{'}(\rho_0)) + \lambda(1-\alpha))^2} \tag{16}$$

When z_2 <0 then it become unstable for uniform steady-state for long-wavelength waves. Thus, the neutral stability condition is given by

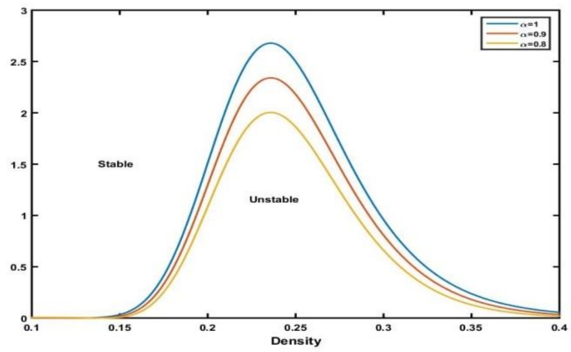


Figure 1: Phase diagram in density-sensitivity with Misconception effect of driver behaviour

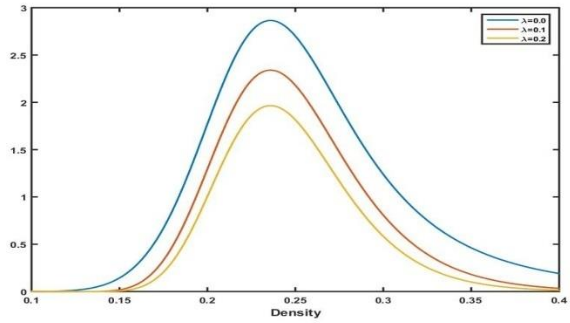


Figure 2: Phase diagram in density-sensitivity with α = 0.9 and λ = 0.0,0.1,0.2

Eq15 Illustrates the importance of the misconception behaviour coefficient (α) playing a pivotal role in stabilizing traffic flow.

In fig 1, the amplitude of the neutral stability curve decrease for the value of α =1.0,0.9 & 0.8 for λ =0.1. Here α is coefficient of Misconception effect of driver behaviour so with decrease the value for α the stable region is increase as shown in fig 1.

In fig 2, the amplitude of the neutral stability curve decrease for the value of λ =0.0,0.1 & 0.2 for α =0.9.Here λ is coefficient of feedback control coefficient so with increase the value for λ the stable region is increase as shown in fig 2.

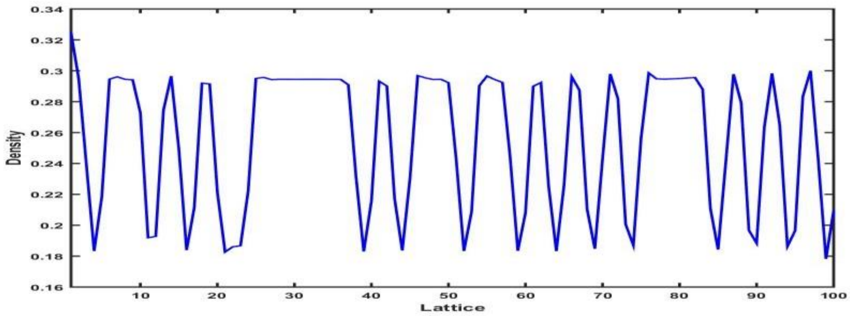


Figure 3: Density profiles at time t=12400 and λ = 0.1 when a = 2.3 and α = 1.0

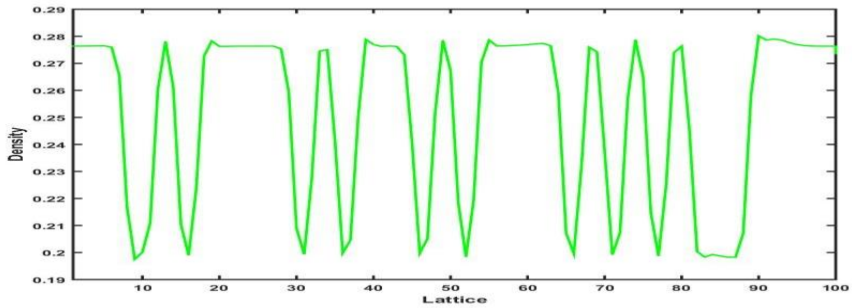


Figure 4: Density profiles at time t=12400 and λ = 0.1 when a = 2.3 and α = 0.9

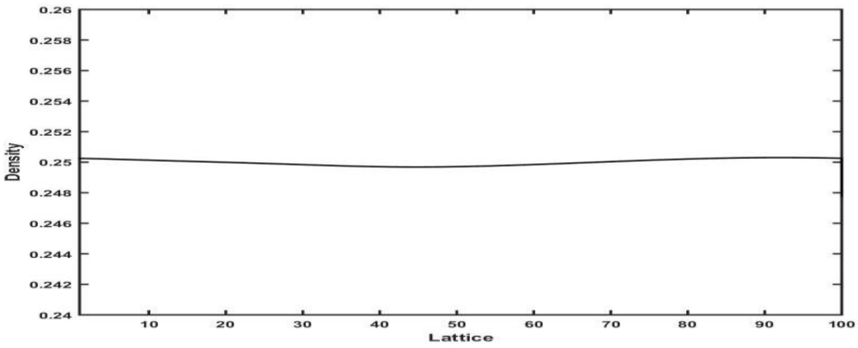


Figure 5: Density profiles at time t=12400 and λ = 0.1 when a = 2.3 and α = 0.8

4. Nonlinear

To investigating the nonlinear stability of the proposed model at the critical point (ρ_0, a_c) involves the use of a reductive perturbation approach. Employing a long-wavelength expansion, this method analyses the gradual shifting behaviour near the critical point and establishes slow scales for spatial and temporal variables.

Listed below are the slow variables X and T. In the context of a small positive parameter ε, these slow variables X and T can be elucidated as:

$$X = \varepsilon(j + bt),\ T = \varepsilon^3 t \qquad (17)$$

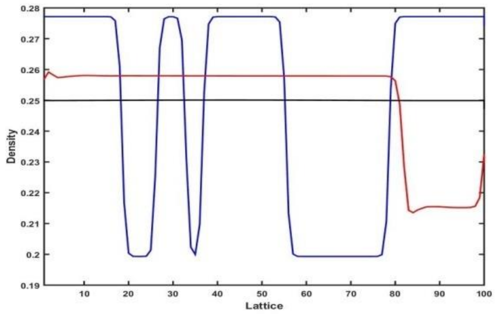


Figure 6: Density at time t=12400 when λ=0.2,0.1 and 0.0 for a=2.3,when α = 0.9

Where b is constant to be determined. Let ρ_j satisfy the following equation:

$$\rho_j(t) = \rho_c + \varepsilon R(X,T) \qquad (18)$$

$$V' = \frac{dV(\rho)}{d\rho} \text{ and } V''' = \frac{d^3V(\rho)}{d(\rho^3)} \text{ at } \rho = \rho_c \qquad (18,a)$$

By expanding Eq.(6) upto the fifth order of ε with the use of Eq.(17), Eq.(18), Eq.(18,a) following nonlinear PDE(partial differential equation) at neighbourhood of critical point τ_c define $\tau = \tau_c(1+\varepsilon^2)$ and choosing

$b = \rho_0^2 V'(\rho_0)$. Eliminating second and third order terms of ε obtained that

$$\varepsilon^4(\partial_T R - g_1\partial_X^3 R + g_2\partial_X R^3) + \varepsilon^5(g_3\partial_X^2 R + g_4\partial_X^4 R + g_5\partial_X^2 R^3) = 0$$

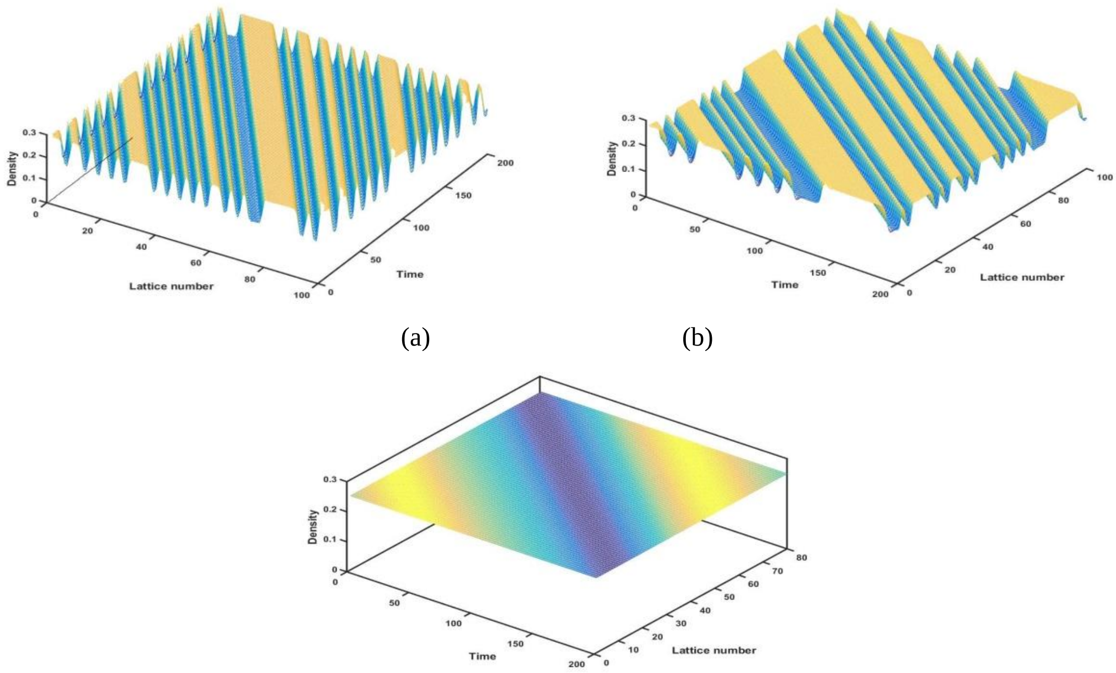
Where g_1,g_2,g_3,g_4 and g_5 as follows

$$g_1 = -(\frac{7}{6}b^3\tau_c^2 + \frac{\alpha\rho_c^2 v'}{6} + \frac{\lambda(1-\alpha)}{6}) \qquad (19)$$

$$g_2 = \frac{\rho_c^2 v'''}{6} \qquad (20)$$

$$g_3 = \frac{3}{2}b^2\tau_c \qquad (21)$$

$$g_4 = 3b\tau_c g_1 + \frac{15}{24}b^4\tau_c^3 + \frac{15}{24}(\alpha\rho_c^2 v') - \frac{1}{24}\lambda(1-\alpha) \qquad (22)$$



(a) (b)

(c)

Figure 7: Spatiotemporal evolutions of density at time t=10300 when a = 2.3 when (a)α =1(b) α =0.9 and (c) α =0.8

$$g_5 = \frac{\alpha \rho_c^2 V'''}{12} - (3b\tau_c)g_2 \tag{23}$$

Now derive the standard mKdV equation, by following transformations have been in used in Eq. (18):

$$T' = g_1 T, R = \sqrt{\frac{g_1}{g_2}} R'$$

$$\partial_T R' - \partial_X^3 R' + \partial_X^3 R'^3 + \varepsilon M[R'] = 0 \tag{24}$$

Where $M[R'] = \frac{1}{g_1}(g_3 \partial_X^2 R' + \frac{g_1 g_5}{g_2} \partial_X^2 R'^3 + g_4 \partial_X^4 R')$

To left the O(ε)terms in Eq. (24),The standard modified Korteweg-de Vries (mKdV) equation is derived, yielding the sought-after kink-soliton solution.

$$R_0'(XT') = \sqrt{c} \tanh \sqrt{\frac{c}{2}}(X - cT') \tag{25}$$

$$(R_0', M[R_0']) = \int_{-\infty}^{\infty} dX R_0', M[R_0'] = 0 \tag{26}$$

with $M[R_0'] = M[R']$ By solving Eq. (26), the value of c is

$$c = \frac{5g_2g_3}{2g_2g_4 - 3g_1g_5}. \tag{27}$$

Hence, the kink-antikink solution is given by

$$\rho_j(t) = \rho_c + \varepsilon \sqrt{\frac{g_1 c}{g_2}} \tanh\left(\sqrt{\frac{c}{2}}(X - cg_1 T\right), \tag{28}$$

with $\varepsilon^2 = \frac{\tau}{\tau_c} - 1$ and the Height A of the solution is

$$A = \sqrt{\frac{g_1}{g_2}\varepsilon^2 c}. \tag{29}$$

Corresponding to nonlinear analysis, these are referred to as coexisting curves and derived the mKdV equation. The kink–antikink solution symbolizes the coexistingphase, encompassing both the freely-moving and congested phases, and can be characterized by $\rho_j = \rho_c \pm A$ respectively in the phase space

(ρ, a). To maintain inclusivity and clarity, it's important to incorporate the misconception effect, which significantly contributes to stabilizing a perfectly smooth traffic flow, without limiting generality.

5. Numerical Simulation

In this phage theoretical results is carried out for the new model with periodicboundary conditions. The initial conditions are given as follows:

$$\rho_j(0) = \rho_j(1) = \begin{cases} \rho_0, \text{ if } j \neq \frac{L}{2}, \frac{L}{2}+1 \\ \rho_0 - \sigma, \text{ if } j = \frac{L}{2} \\ \rho_0 + \sigma, \text{ if } j = \frac{L}{2}+1 \end{cases}$$

In this context, σ denotes the initial disturbance, while L represents the total number of sites, fixed at 100, with other parameters set as follows: $\rho_0 = 0.1, \tau = \frac{1}{a}$

For computational accuracy, the maximal velocity and critical density are defined as 2 and 0.25, respectively.

Notably, in figure3 and figure4 there is small perturbation for α values of 1, and 0.9 with in the stable region, and for α = 0.8 figure5 presents the initial perturbation diminishes of density evolution after 10^4 time steps for the fixed value for λ = 0.1 resulting in a uniform traffic flow. Furthermore, with an increase to α = 0.1, the number and height of stop-and-go waves increase, leading to congestion, as evident in figure3,figure4, figure5.

Similarly, in figure6,the simulation results for density evolution after 10^4 time steps with different λ values are displayed for and fixed α = 0.8. As λ decreases, the uniform flow transforms into congestion with kink–antikink-soliton density waves in the unstable region as illustrated in figure6.

In figure7 the simulation results for density evolution after 10^4 time steps with Spatiotemporal evolutions of density at time t=10300 when a = 2.3 with (a) α = 1 (b) α = 0.9 and (c) α = 0.8

Comparing results for the values of α , and λ it is concluded that the effect of misconception behaviour significantly impacts on smooth traffic flow. In our findings, a decrease in the value of α corresponds to an increase in stability.

After simulation our findings as follows:

The simulation results from our investigation into the influence of driver misconceptions on road behaviour using a novel lattice hydrodynamic model provide valuable insights into the complex dynamics of traffic flow. Through rigorous experimentation and analysis, we have uncovered several key findings that shed light on the interplay between cognitive biases and traffic patterns.

One notable outcome of our simulations is the identification of distinct traffic regimes characterized by varying levels of congestion and stability. By incorporating driver misconceptions into our model, we observed shifts in these regimes, with cognitive biases exacerbating congestion in certain scenarios while mitigating it in others. This highlights the nuanced and context-dependent nature of the impact of driver cognition on road behaviour.

Furthermore, our simulations revealed the role of driver misconceptions in shaping the emergence of traffic flow patterns such as phantom jams, stop-andgo waves, and lane-changing behaviours. By capturing the cognitive biases that influence individual driver decisions, our model provided a more comprehensive understanding of these phenomena and their underlying mechanisms.

Additionally, our simulations allowed us to assess the effectiveness of potential interventions aimed at mitigating the negative effects of driver misconceptions on road behaviour. By implementing targeted strategies such as improved signage, driver education programs, or adaptive traffic management systems, we were able to observe changes in traffic flow dynamics and congestion levels, demonstrating the potential for practical interventions to address cognitive biases in real-world traffic scenarios.

Overall, the simulation results from our study underscore the importance of considering driver misconceptions in traffic modelling and management efforts. By incorporating cognitive factors into modelling frameworks, we can develop more accurate predictive models, design more effective interventions, and ultimately improve the efficiency, safety, and sustainability of our transportation systems.

6. Conclusion

In conclusion, our in-depth exploration of the impact of driver misconceptions on road behaviour has highlighted the multifaceted nature of traffic dynamics and the pivotal role of human factors in shaping transportation systems. By leveraging innovative modelling approaches such as the lattice hydrodynamic framework we have advanced our understanding of these phenomena and laid the groundwork for more nuanced and effective strategies for smooth traffic flow and safety enhancement. Moving forward, further research in this domain promises to yield even greater insights and contribute to the development of more resilient and adaptive transportation systems. This study reveals that drivers who possess awareness of traffic rules and maintain composure are capable of driving smoothly. The findings of this research directly impact the occurrence of traffic congestion and accidents.

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