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Research Article

An Eoq Model For Imperfect Quality Deteriorating Items With Stock-Dependent Demand & Learning Effect Under Two-Level Trade Credit Financing Policy Subject To Partial Backlogging

R.Sandhya[1*], Dr.D.Chitra[2]

[1*]Research Scholar, PG & Research Department of Mathematics, Quaid-E-Millath Government College for Women Autonomous, Anna salai, Chennai-02, Tamilnadu, India. E-mail: sandhyaraja1988@gmail.com
[2] Associate Professor, PG & Research Department of Mathematics, Quaid-E-Millath Government College for Women Autonomous, Anna salai, Chennai-02, Tamilnadu, India. E-mail: dchitraravichandran1972@gmail.com

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ARTICLE INFO	ABSTRACT

In this article an EOQ model for imperfect quality deteriorating items with stock-dependent demand is developed. It is assumed that the supplier has perfect and imperfect quality items. When the supplier provides lots for sale to his retailer, the retailer separates the whole lot by inspection process into perfect and imperfect quality items. The percentage of defective items present in the lot follows S-shape learning curve. Retailer will avail a price discount from the supplier due to rework process and customers will get a price discount for reworked items. The retailer who purchases the items enjoys a fixed credit period offered by the supplier and in turn, also offers a credit period to the customer in order to compete the market competition. Shortages are allowed and are partially backlogged. The backlogging rate varies inversely as the waiting time for the next replenishment. The main objective of the inventory model is to minimize the total cost per unit time and to determine the optimal cycle length. Numerical examples have given to elucidate our model. Finally sensitivity analysis has been carried for variation of the input parameters on the decision variables and total cost.

Keywords: Deteriorating items, Imperfect Quality items, Learning curve, Stock-dependent demand, partial backlogging, Two-Level credit period.

1. INTRODUCTION

Inventory control is essential for companies to reduce their costs, maintaining stock, improving products quality, providing better services and managing customer demands, companies are facing greater challenges when they are working with deteriorating products. But in real life there are numerous products like dry fruits, food grains, fresh fruits, vegetables, milk, meat, medicine, volatile liquids, and blood banks etc., that have a shelf-life and start deteriorating after some time. This underlines the fact that for some initial period of time, there is no deterioration in items. This phenomenon is termed as non-instantaneous deterioration and the items are termed as non-instantaneous deteriorating items. Large quantity of goods displayed in market attract the customers to buy more. If the stock is insufficient, customer may prefer some other brand, as a result it will fetch loss to the supplier. In some inventory systems such as fashionable commodities the length of waiting time for the next replenishment is the main factor in determining whether backlogging will be accepted or not, the longer the waiting time is the smaller the backlogging rate would be and vice versa.
The nature of demand depends on many factors viz., selling price, availability of the stock, time, quality of the product, ecofriendliness of the product, impreciseness, etc. The traditional literature on inventory models assumes that the demand of the items is uniform on time, and independent of the levels of stock for sale in the warehouse. However, for some types of products, it is a proven fact that the quantity in stock can influence the item sales. Low stocks of certain items make customers feel that they are not fresh and drastically reduce sales of those items. Conversely, increasing the amount of the article displayed for sale

induces more consumers to buy it. This probably occurs owing to the increase in the visibility of the items. Hence, inventory managers have observed that the demand rate may be influenced by the stock levels. Thus, in inventory control, the possible effect of the inventory level on the demand has been recognized and studied by some researchers. As a starting point, Wolfe [1] showed empirical evidence that sales of style merchandise are almost proportional to the displayed inventory. In addition, the book by Levin et al. [2] mentioned that the presence of inventory has a motivational effect, as evidenced by the issue that large piles of goods displayed in a supermarket will lead the customers to buy more. Later, Silver and Peterson [3] confirmed the results obtained by Wolfe that sales at the retail level tend to be directly proportional to the displayed stock. Larson and DeMarais [4] investigated the impact of displayed inventory on sales for four health and beauty items and offered possible explanations for the effect of stock level on demand. They introduced the term "psychic shock" to refer to the reasons for this effect. Achabal et al. [5] provided empirical evidence that displayed inventory increases the demand of goods. The adventage of using a stock-dependent demand was highlighted by Balakrishnan et al. [6]. In addition, Koschat [7] analyzed a real case in the magazine industry, proving that demand can indeed vary with inventory level. Thus, an inventory drop for one brand leads to a decrease of demand for that brand and in an increase of demand for a competing brand. Next, we present a revision of the literature on inventory models with this type of demand.

Baker and Urban [8] developed a first deterministic inventory system for a stockdependent demand rate. Padmanabhan and Vrat [9] analyzed an inventory system for deteriorating multi-items with stock-dependent demand. Datta and Pal [10] established the optimal inventory policy for a system where the demand rate is constant if the inventory level is less than a given level, and this rate depends on the instantaneous inventory level when it is greater than that given level. Later, Urban [11] relaxed the condition of zero-inventory at the end of the inventory cycle considered by Datta and Pal [10]. The model of Baker and Urban [8] was generalized by Pal et al. [12] for items with a constant deterioration rate. Giri et al. [13] also extended the inventory model of Urban [11] in the same way. Giri and Chaudhuri [14] developed the economic order quantity model for stockdependent demand considering a non-linear holding cost. Datta and Paul [15] studied a multi-period inventory system where the demand rate is stock-dependent and sensitive to the selling price. Ouyang [16] introduced a model with stock-dependent demand for deteriorating items under conditions of inflation and time-value of money, considering the present value of the total inventory cost as the objective function. Chang [17] analyzed an inventory model with non-linear holding cost, where the demand rate depended on the stock level. Later, Pando et al. [18–20] considered three models with maximization of the profit per unit time and stock-dependent demand, where the holding cost was non-linear on time, on the stock level or, even more, on both quantities. Yang [21] presented an inventory model where the demand rate and the holding cost rate are stock-dependent, and partially backlogged shortages. Annadurai and Uthayakumar [22] described a lot-sizing model for deteriorating items with stock-dependent demand, partially backordered shortages and delay in payments. In the same line, Choudhury et al. [23] analyzed an inventory system with shortages and time-varying holding costs.

Permissible delay in payment is one of the most important and popular strategies in the business world. With this strategy, the suppliers attract more customers, and this leads to more sales of the product. The retailers then accept this and also give the facility of a delay in payment (credit time) to his/her customers. The delay in payment (credit time) facility consists of giving to a buyer the payment facility of paying the purchased amount up to a certain period. On the one hand, no interest is charged by the vendor to his/her buyer if the payment is done up to the credit period. On the other hand, an interest is charged by the vendor to his/her buyer when the credit period is over. Recently, several researchers have performed several kinds of research in twolevel trade credit concept. Teng [24] introduced an inventory model with trade credit financing by distinguishing the difference between unit cost and unit price. Thereafter, Huang [25] extended the inventory model with the trade credit for a supply chain system with both an upstream and a downstream credit. Teng and Goyal [26] modified the assumption of Huang's inventory model by introducing the concept that the retailer obtains its revenue from N to N + T, not from 0 to T. At the same time, Teng [27] obtained the optimal ordering policy for the retailer to deal with credit risk clients, as well as good credit clients. Min et al. worked on an inventory model under stock dependent demand and two-level trade credit. In this line of research, there exist the following papers in this area, Kreng and Tan [28], Teng and Lou [29] Mahata [30], Chung and Cárdenas-Barrón [31], Chung et al. [32], Chen et al. [33], Shah and Cárdenas-Barrón [34] and others. For example, Jaggi et al. Aliabadi et al. [35] solved a non-instantaneous deteriorating inventory problem with credit period and carbon emissions dependent demand by using a geometric programming approach. Liao et al. [36] addressed an EOQ inventory model with a delay in payment policy for non-instantaneous deteriorating items with the aim of finding an optimal ordering policy. Shaikh et al. [37] considered price-sensitive demand, inflation, and reliability. Recently Abu Hashan Md Mashud [38] worked on integrated price-sensitive inventory model for deteriorating items under trade credit policy.

The effects of learning play a vital role for reducing the inventory cost and optimizing the total profit of the inventory model. Some authors discussed the results of the learning shape in the same direction, such as Wright [39] and Jaber et al. [40]. From Figure 1, it can be observed that the curve rises slowly as one becomes familiar with the basics of a skill. The steep part occurs when one has enough experience to start "putting it all together." Then, a second phase of fast development is entered, known as Learning Phase 2. Skills are added along with the progress. At a certain moment, development achieves a speed of steady development, followed by a period of slower development. The final phase of top of the progress is known as maturity phase. In Figure 2, the S-shape learning curve is graphically represented with the help of the available data which is provided below in the form of a formula: $P(n)=\dfrac{d}{f+e^{gn}}$ where d, f and $g>0$ are the model parameters, n is the cumulative number of cycle and with $P(n)$ is the percentage of defective items per cycle n. Fig 1: Learning Stage Fig 2: S-Shape Learning C

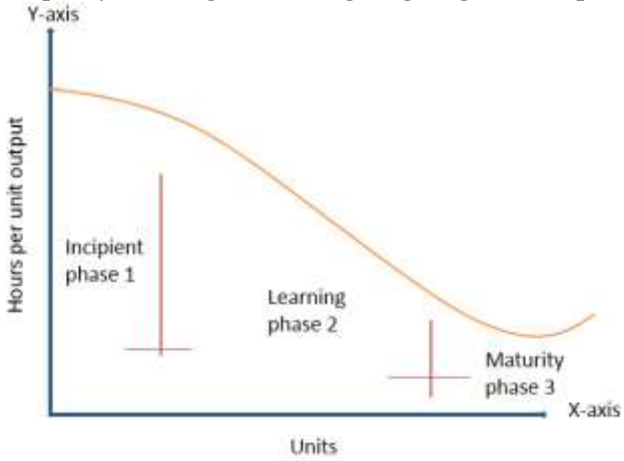


Fig 1: Learning Stage

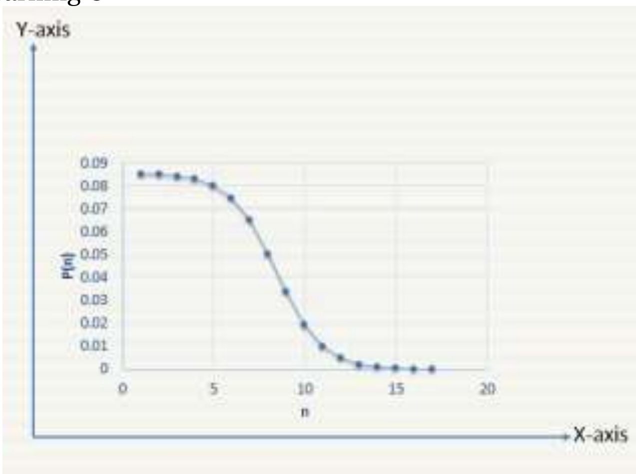


Fig 2: S-Shape Learning Curve

In this work, an effort has been made to analyze an EOQ model for imperfect quality deteriorating items with stock-dependent demand under learning effect. Also two-level trade credit policy is considered, in which, the supplier offers a trade credit facility to the retailer and then retailer also provides a trade credit to the customers. In addition shortages are allowed and partially backlogged. The proposed model is formulated mathematically by using ordinary differential equations and the corresponding optimization is obtained as a cost minimization problem.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations have been used in the entire paper.

2.1 ASSUMPTIONS:

1. Demand rate is known and which is stock dependent. The Consumption rate $D(t)$ at time t is assumed to be

$$D(t)=\begin{cases} a+bI(t), 0\le t\le t_1 \\ a, t_1\le t\le T \end{cases}$$

Where a and b are positive constants.

2. Shortages are allowed and the backlogged rate is defined to be $\dfrac{1}{1+\delta(T-t)}$ when inventory is negative. The backlogging parameter δ is a positive constant.
3. The replenishment rate is infinite and the lead time is zero. The time horizon is infinite.
4. Both screening as well as demand proceeds simultaneously, but the screening rate is assumed to be greater than demand rate.
5. Imperfect quality items follow the S-shape learning curve with $P(n)=\dfrac{d}{f+e^{gn}}$ where d, f and $g>0$ are the model parameters, n is the cumulative number of cycle and $P(n)$ is the percentage of defective items per cycle n.
6. The fixed credit period offered by the supplier to the retailer is no less than the credit period permitted by the retailer to the customer (ie) $M\ge N$
7. When $T\le M$, the account is settled at $t=M$ and the retailer does not need to pay any interest charge of items in stock during the whole cycle.
8. The retailer can accumulate revenue and earn interest during the period from $t=N$ to $t=M$ with rate I_e under the condition of trade credit.

2.2 NOTATIONS:

$I(t)$	-	Inventory level at any time t where $0 \le t \le T$
$D(t)$	-	Demand rate function
θ	-	Deterioration rate, $0 < \theta < 1$.
P_1	-	The purchasing cost per perfect quality item ($/unit)
P_2	-	The purchasing cost per defective item ($/unit)
c_1	-	The selling price per perfect quality item ($/unit)
c_2	-	The selling price per defective item ($/unit)
c_s	-	The screening cost per unit item ($/unit)
K	-	The Replenishment cost per order ($/order)
h	-	Holding cost per unit per unit time
s	-	Shortage cost per unit time ($/unit)
π	-	Opportunity cost due to lost sale per unit time ($/unit)
δ	-	The Backlogging parameter (a positive constant) $0 < \delta < 1$.
$P(n)$	-	Percentage of defective items
μ	-	Screening rate per unit item
t_1	-	Time at which shortages starts, $0 \le t_1 \le T$.
T	-	The length of the Replenishment cycle
S	-	Maximum product amount at the very beginning of the cycle
R	-	Maximum storage amount at the end of the cycle
M	-	The retailer's trade-credit period offered by supplier
N	-	The customer's trade-credit period offered by retailer
I_e	-	Interest which can be earned per $ per year by retailer
I_r	-	Interest charged per $ in stocks per year by the supplier
$TC(t_1,T)$	-	The total cost per unit time, $ per unit time.

3. MATHEMATICAL FORMULATION:

Based on the assumptions mentioned earlier, this section presents the following inventory model formulation. In the beginning, an enterprise purchase Q units. It is assumed that each lot received contains percentage defectives $\alpha = P(n)$ which follows S-Shape learning curve. Screening (inspection) process of the whole received lot is undertaken at a rate of μ units per unit time; During this period the demand, deterioration and screening process occurs simultaneously and demand is fulfilled from the items which are found to be of perfect quality by the screening process and the defective items are used to eliminate backorders and sold immediately after the screening process at time t_s as a single batch with a price discount. Shortages are allowed and partially backlogged which are eliminated during the screening process as it has been assumed that screening rate is greater than the demand rate. The behavior of the inventory level is illustrated in Fig. 1, where T is the cycle length, αQ is the number of defectives withdrawn from inventory, $t_s = \dfrac{Q}{\mu}$ is the total screening time of Q units ordered per cycle which is less than the cycle time T and t_1 is the inventory level reaches zero respectively.

Therefore, the inventory system is presented with the help of the following differential equations:

$$\frac{dI_1(t)}{dt} = -a - bI_1(t) - \theta I_1(t), 0 \le t \le t_1$$

$$\frac{dI_2(t)}{dt} = -\frac{a}{1+\delta(T-t)}, t_1 \le t \le T \tag{1}$$

with boundary conditions $I(t_1) = 0$, and $I(t)$ is continuous at $t = t_1$.

The solutions of the above differential equation are

$$I_1(t) = -\frac{a}{b+\theta} + \frac{e^{-(b+\theta)t}a}{e^{-(b+\theta)t_1}(b+\theta)} \quad (2)$$

$$I_2(t) = \frac{a\left(\log[1+\delta(T-t)] - \log[1+\delta T - \delta t_1]\right)}{\delta} \quad (3)$$

Exploiting the boundary condition $I_1(0) = S$, we get

$$S = -\frac{a}{b+\theta} + \frac{a}{e^{-(b+\theta)t_1}(b+\theta)} \quad (4)$$

Again exploiting the boundary condition $I_2(T) = -R$, we get

$$R = \frac{a\log[1+\delta T - \delta t_1]}{\delta} \quad (5)$$

The total number of ordered amount is

$$Q = S + R$$

$$= -\left[\frac{a}{b+\theta} + \frac{a}{e^{-(b+\theta)t_1}(b+\theta)} + \frac{a\log(1+\delta T - \delta t_1)}{\delta}\right] \quad (6)$$

The total inventory cost consists of the following components.

a) Ordering cost per cycle is K. (7)

b) Inventory holding cost HC per cycle is given by

$$HC = h\int_0^{t_1} I_1(t)dt$$

$$= \frac{ha\left(e^{t_1(b+\theta)} - 1 - t_1 b - t_1\theta\right)}{b^2 + 2b\theta + \theta^2} \quad (8)$$

c) The shortage cost in the interval $[t_1, T)$ denoted by SC is given by,

$$SC = s\int_{t_1}^{T} -I_2(t)dt$$

$$= -\left\{\frac{sa\left[\log(1+\delta T - \delta t_1) - \delta T + \delta t_1\right]}{\delta^2}\right\} \quad (9)$$

d) The opportunity cost due to lost sales denoted by OC is given by,

$$OC = \pi\int_{t_1}^{T} \alpha(p)\left[1 - \frac{1}{1+\delta(T-t)}\right]dt$$

$$= \pi aT - \pi\left[at_1 + \frac{a\log(1+\delta T - \delta t_1)}{\delta}\right] \quad (10)$$

e) The Purchasing cost for the first replenishment cycle denoted by PC is given by

$$PC = P_1(1-\alpha)Q + P_2\alpha Q$$

$$PC = P_1(1-\alpha)\left[-\frac{a}{b+\theta} + \frac{a}{e^{-(b+\theta)t_1}(b+\theta)} + \frac{a\log(1+\delta T - \delta t_1)}{\delta}\right]$$

$$+ P_2\alpha\left[-\frac{a}{b+\theta} + \frac{a}{e^{-(b+\theta)t_1}(b+\theta)} + \frac{a\log(1+\delta T - \delta t_1)}{\delta}\right]$$

$$PC = -\frac{1}{(b+\theta)e^{-(b+\theta)t_1}\delta}\left\{\left[(P_1 - P_2)\alpha - P_1\right]\left[e^{-(b+\theta)t_1}(b+\theta)\log(1+\delta(T-t_1))\right] - \delta\left[e^{-(b+\theta)t_1} - 1\right]a\right\} \quad (11)$$

f) Sales revenue is the sum of revenue generated by the demand meet during the time period $(0,T)$ and by the sale of imperfect quality items is $c_1(1-\alpha)Q + c_2\alpha Q$

$$SR = c_1(1-\alpha)\left[-\frac{a}{b+\theta} + \frac{a}{e^{-(b+\theta)t_1}(b+\theta)} + \frac{a\log(1+\delta T-\delta t_1)}{\delta}\right]$$

$$+c_2\alpha\left[-\frac{a}{b+\theta} + \frac{a}{e^{-(b+\theta)t_1}(b+\theta)} + \frac{a\log(1+\delta T-\delta t_1)}{\delta}\right]$$

$$SR = -\frac{1}{(b+\theta)e^{-(b+\theta)t_1}\delta}\left\{\left[(c_1-c_2)\alpha - c_1\right]\left[e^{-(b+\theta)t_1}(b+\theta)\log(1+\delta(T-t_1))\right] - \delta\left[e^{-(b+\theta)t_1}-1\right]a\right\} \tag{12}$$

g) Screening cost c_sQ

$$SCR = ac_s\frac{\left[e^{-(b+\theta)t_1}(b+\theta)\log(1+\delta(T-t_1)) - \delta\left(e^{-(b+\theta)t_1}-1\right)\right]}{(b+\theta)e^{-(b+\theta)t_1}\delta} \tag{13}$$

Therefore the total inventory cost (X) = K − Sales revenue + Holding cost + Shortage cost + Opportunity cost + Purchasing cost + Screening Cost

i.e. $X = K -$ Sales Revenue + Purchase Cost + Screening Cost + $HC + SC + OC$

$$= \left\{\begin{array}{l} K - \frac{1}{(b+\theta)e^{-(b+\theta)t_1}\delta}\left\{\left[(c_1-c_2)\alpha - c_1\right]\left[e^{-(b+\theta)t_1}(b+\theta)\log(1+\delta(T-t_1))\right] - \delta\left[e^{-(b+\theta)t_1}-1\right]a\right\} \\ + -\frac{1}{(b+\theta)e^{-(b+\theta)t_1}\delta}\left\{\left[(P_1-P_2)\alpha - P_1\right]\left[e^{-(b+\theta)t_1}(b+\theta)\log(1+\delta(T-t_1))\right] - \delta\left[e^{-(b+\theta)t_1}-1\right]a\right\} \\ + ac_s\frac{\left[e^{-(b+\theta)t_1}(b+\theta)\log(1+\delta(T-t_1)) - \delta\left(e^{-(b+\theta)t_1}-1\right)\right]}{(b+\theta)e^{-(b+\theta)t_1}\delta} \\ + \frac{ha\left(e^{t_1(b+\theta)} - 1 - t_1 b - t_1\theta\right)}{b^2+2b\theta+\theta^2} + -\left\{\frac{sa\left[\log(1+\delta T-\delta t_1) - \delta T + \delta t_1\right]}{\delta^2}\right\} \\ + \pi a T - \pi\left[at_1 + \frac{a\log(1+\delta T-\delta t_1)}{\delta}\right] \end{array}\right\} \tag{14}$$

Considering the permissible delay period M for supplier offered by the Retailer and N for customer offered by the supplier, the inventory model has the following cases

Case 1: $M \le t_1$

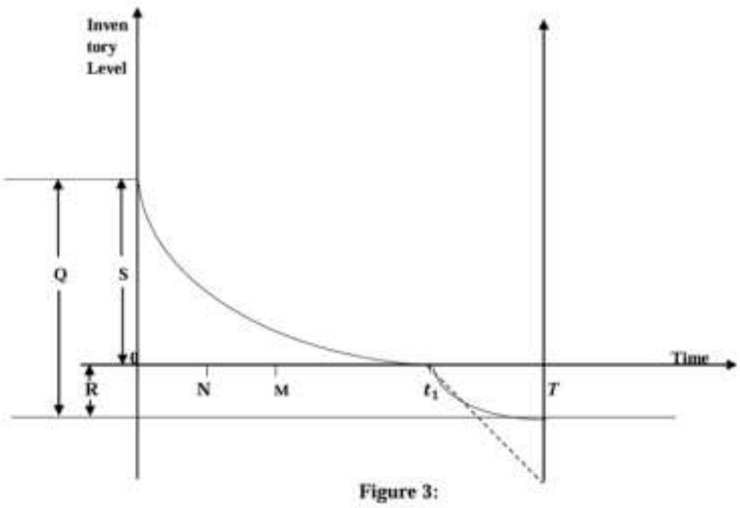


Figure 3:

This case can be extended as $0 \le N \le M \le t_1$, in which both N and M is shorter than t_1. The interest payable in this case is

$$IP_{11} = P_1 I_r \left(\int_M^{t_1} I_1(t)dt \right)$$

$$IP_{11} = \frac{1}{(b+\theta)^2} \left\{ a\left[\left((M-t_1)\theta - 1 + (M-t_1)b \right) e^{-(b+\theta)t_1} + e^{-(b=\theta)M} \right] e^{(b+\theta)t_1} P_1 I_r \right\}$$

(15)

The interest earned in this case is

$$IE_{11} = \left[c_1 I_e \left(\int_N^M (a + bI_1(t))tdt \right) + (c_2 I_e \alpha Q) \right]$$

$$IE_{11} = \begin{Bmatrix} \frac{1}{2}\frac{1}{(b+\theta)^3}\left[c_1 I_e a \left(-N^2b^2\theta - 2N^2b\theta^2 - N^2\theta^3 + 2b^2 e^{t_1b+t_1\theta-Nb-N\theta} N + 2be^{t_1b+t_1\theta-Nb-N\theta} N\theta \right.\right. \\ + 2be^{t_1b+t_1\theta-Nb-N\theta} + M^2b^2\theta + 2M^2b\theta^2 + M^2\theta^3 - 2b^2e^{t_1b+t_1\theta-Mb-M\theta}M - 2be^{t_1b+t_1\theta-Mb-M\theta}M\theta \\ \left.\left. - 2be^{t_1b+t_1\theta-Mb-M\theta} \right)\right] + c_2 I_e \alpha \left(-\frac{a}{b+\theta} + \frac{a}{e^{-(b+\theta)t_1}(b+\theta)} + \frac{a\log(1+\delta T - \delta t_1)}{\delta} \right) \end{Bmatrix}$$

(16)

Case 2: $M \ge t_1$

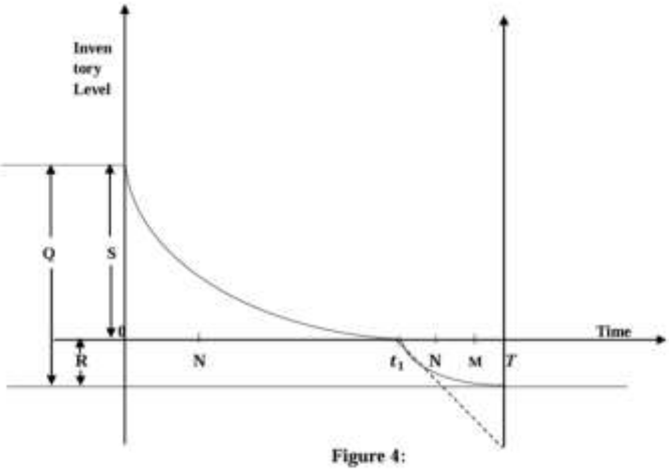


Figure 4:

In this case, two sub-cases may arise, $0 \le N \le t_1 \le M$ and $0 \le t_1 \le N \le M$

Subcase 21: $0 \le N \le t_1 \le M$

The interest payable in this case is 0

$$IP_{21} = 0 \qquad (17)$$

The interest earned in this case is

$$IE_{21} = c_1 I_e \left(\int_N^{t_1} (a + bI_1(t))tdt + \int_{t_1}^M (a + bI_2(t))tdt \right) + c_2 I_e \alpha Q$$

$$IE_{21}=\left\{\begin{array}{l} c_1I_e\left[-\frac{1}{2}\frac{1}{(b+\theta)^3}\left(a\left(N^2b^2\theta+2N^2b\theta^2+N^2\theta^3-2b^2e^{t_1b+t_1\theta-Nb-N\theta}N-2be^{t_1b+t_1\theta-Nb-N\theta}N\theta\right.\right.\right. \\ \left.\left.-2be^{t_1b+t_1\theta-Nb-N\theta}-b^2\theta t_1^2-2b\theta^2t_1^2-\theta^3t_1^2+2bt_1\theta+2b+2b^2t_1\right)\right)+\frac{1}{4}\frac{1}{\delta^3}\left(a\left(-2b\log(1+\delta T-\delta M)\right.\right. \\ -2b\log(1+\delta T-\delta M)T^2\delta^2+2b\log(1+\delta T-\delta M)M^2\delta^2-4b\log(1+\delta T-\delta M)T\delta \\ +2b\log(1+\delta T-\delta t_1)T^2\delta^2-2b\log(1+\delta T-\delta t_1)M^2\delta^2+4b\log(1+\delta T-\delta t_1)\delta T+2b\log(1+\delta T-\delta t_1) \\ \left.\left.\left.+2b\delta t_1+b\delta^2t_1^2-2\delta^3t_1^2+2bT\delta^2t_1+2M^2\delta^3-2bTM\delta^2-2bM\delta-bM^2\delta^2\right)\right)\right] \\ +c_2I_e\alpha\left[-\frac{a}{b+\theta}+\frac{a}{e^{-(b+\theta)t_1}(b+\theta)}+\frac{a\log(1+\delta T-\delta t_1)}{\delta}\right] \end{array}\right\}$$

(18)

Subcase 22: $0\le t_1\le N\le M$

The interest payable in this case is 0

$$IP_{22}=0$$

The interest earned in this case is

$$IE_{22}=c_1I_e\left(\int_N^M(a+bI_2(t))tdt\right)+(c_2I_e\alpha Q)$$

$$IE_{22}=\left\{\begin{array}{l} -\frac{1}{4}\frac{1}{\delta^3}\left(c_1I_ea\left(-2bN\delta-bN^2\delta^2-2b\log(1+\delta T-N\delta)+2bM\delta+bM^2\delta^2\right.\right. \\ +2b\log(1+\delta T-M\delta)+2N^2\delta^3-2M^2\delta^3-2bTN\delta^2-2b\log(1+\delta T-N\delta)T^2\delta^2 \\ +2b\log(1+\delta T-N\delta)N^2\delta^2-2b\log(1+\delta T-\delta t_1)N^2\delta^2-4b\log(1+\delta T-N\delta)T\delta \\ +2bTM\delta^2+2b\log(1+\delta T-M\delta)T^2\delta^2-2b\log(1+\delta T-M\delta)M^2\delta^2+2b\log(1+\delta T-\delta t_1)M^2\delta^2 \\ \left.\left.+4b\log(1+\delta T-M\delta)T\delta\right)\right)+c_2I_e\alpha\left[-\frac{a}{b+\theta}+\frac{a}{e^{-(b+\theta)t_1}(b+\theta)}+\frac{a\log(1+\delta T-\delta t_1)}{\delta}\right] \end{array}\right\}$$

(19)

Total Cost Function:

$$TC_i(T,t_1)=\begin{cases} TC_1, 0\le N\le M\le t_1 \\ TC_2, 0\le N\le t_1\le M \\ TC_3, 0\le t_1\le N\le M \end{cases} \quad (20)$$

$$TC_1(T,t_1)=\frac{1}{T}[X+IE_{11}-IP_{11}] \quad (21)$$

$$TC_2(T,t_1)=\frac{1}{T}[X+IE_{21}-IP_{21}] \quad (22)$$

$$TC_3(T,t_1)=\frac{1}{T}[X+IE_{22}-IP_{22}] \quad (23)$$

The necessary conditions for the total cost $\partial TC_i(T,t_1)$ is convex with respect to T and t_1 are $\frac{\partial TC_i(T,t_1)}{\partial T}=0$ and $\frac{\partial TC_i(T,t_1)}{\partial t_1}=0$

Provided they satisfy the sufficient conditions $\left.\frac{\partial^2TC_i(T,t_1)}{\partial T^2}\right|_{(T^*,t_1^*)}>0, \left.\frac{\partial^2TC_i(T,t_1)}{\partial t_1^{\,2}}\right|_{(T^*,t_1^*)}>0$

and $\begin{bmatrix} \frac{\partial^2 TC_i}{\partial T^2} & \frac{\partial^2 TC_i}{\partial T \partial t_1} \\ \frac{\partial^2 TC_i}{\partial t_1 \partial T} & \frac{\partial^2 TC_i}{\partial t_1^2} \end{bmatrix} > 0$

We develop the following algorithm to find the optimal values of T and t_1 (say T^*, t_1^*) that minimize $TC_i(T, t_1)$

SOLUTION PROCEDURE:

The problem mentioned above is solved by using the following algorithm:

Step1: Start

Step 2: Plug all the value of the required parameters of the proposed model in the equation (20)

Step 3: Put $\frac{\partial TC_i}{\partial t_1} = \frac{\partial TC_i}{\partial T} = 0$, $i = 1,2,3$

Step 4: Solve the optimization problem TC_i for $i = 1,2,3$ and store the optimal value of t_1^*, T^*, TC^*, S^* and R^*

Step 5: Compare the value of TC_1, TC_2 and TC_3.

Step 6: Choose the minimum value among TC_1, TC_2 and TC_3.

Step 7: Stop

NUMERICAL EXAMPLES:

Example 1: Consider the inventory system with the following data $K = 1000, a = 400, b = 2, \alpha = 0.05, \theta = 0.08, \delta = 0.42, h = 2.8, P_1 = 25, P_2 = 15, c_1 = 27, c_2 = 17, cs = 0.3,$ $s = 18, \pi = 13, I_e = 0.12, I_r = 0.13, M = 0.08, N = 0.04$ in appropriate units. In this case $N \le M \le t_1$. Using the algorithm we obtain the optimal solution as $T = 0.5567$, $t_1 = 0.4436$. Hence the Total cost per unit time is $TC_1 = 3053.51, Q = 335.81$.

Example 2: Taking all the parameters same except $M = 0.9, N = 0.2$ in appropriate units. In this case $N \le t_1 \le M$. Using the algorithm we obtain the optimal solution as $T = 0.8190$, $t_1 = 0.6210$. Hence the Total cost per unit time is $TC_2 = 2673..41, Q = 583.69$.

Example 3: Taking all the parameters same except $M = 0.9, N = 0.7$ in this case $t_1 \le N \le M$. Using the algorithm we obtain the optimal solution as $T = 0.5084$, $t_1 = 0.3313$. Hence the Total cost per unit time is $TC_3 = 2459.99, Q = 259.24$.

Effect of change in various parameter of the inventory is presented in the following table

Changing parameter	Change in parameter	t_1	T	TC	Q
K	855 900 1100 1155	0.3157 0.3206 0.3416 0.3476	0.4824 0.4906 0.5257 0.5350	2212.776 2291.097 2621.762 2708.280	243.029 248.057 270.043 275.997
a	300 350 450 500	0.3096 0.3194 0.3460 0.3647	0.4722 0.4886 0.5330 0.5644	2643.534 2555.893 2353.161 2233.261	296.049 277.671 240.350 221.306
θ	0.0684 0.0720 0.0792 0.0831	0.3311 0.3312 0.3313 0.3313	0.5085 0.5084 0.5084 0.5084	2458.936 2459.196 2459.196 2459.716	258.666 258.806 259.088 259.241
b	1.6 1.8 2.2 2.4	0.3151 0.3232 0.3395 0.3477	0.5023 0.5055 0.5111 0.5271	2208.280 2356.068 2406.767 2515.498	238.287 248.170 271.296 284.888
δ	0.359 0.378 0.415 0.435	0.3367 0.3348 0.3317 0.3302	0.5271 0.5208 0.5098 0.5043	2404.450 2421.786 2455.275 2473.236	268.779 265.534 259.835 257.041

M	0.935	0.3391	0.5118	2487.161	263.746
	0.891	0.3294	0.5176	2453.662	258.035
	0.990	0.3527	0.5186	2442.350	272.389
	1.039	0.3663	0.5261	2404.740	281.578
N	0.598	0.3395	0.5022	2439.035	260.326
	0.630	0.3373	0.5045	2448.087	260.273
	0.770	0.3230	0.5099	2457.616	256.177
	0.808	0.3173	0.5093	2459.294	253.652

Graphical Representation of the Effect of Change in Various Parameters of the Inventory:

Fig.5. Effect of Change K on Total Cost

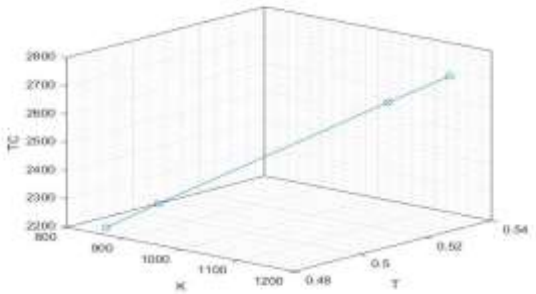


Fig.6. Effect of Change a on Total Cost

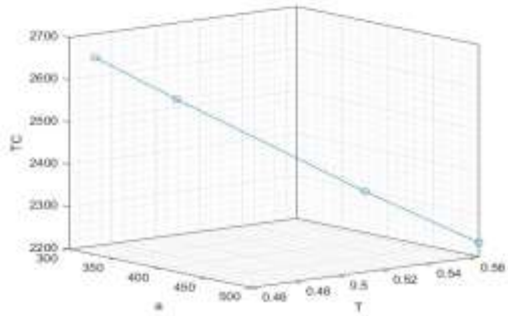


Fig.7. Effect of Change θ on Total Cost

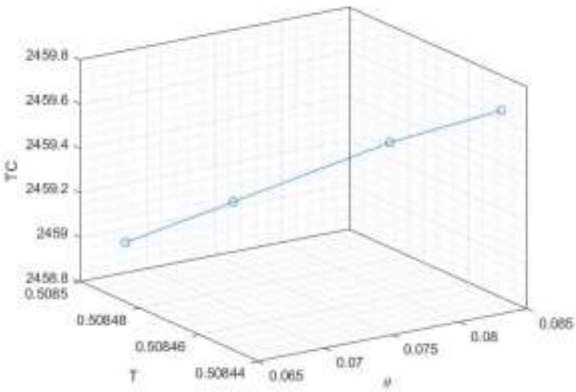


Fig.8. Effect of Change b on Total Cost

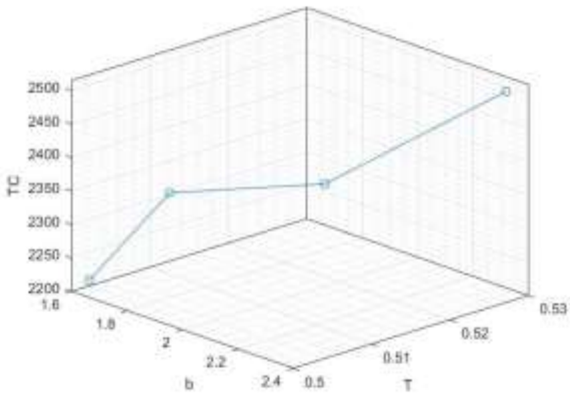


Fig.9. Effect of Change δ on Total Profit

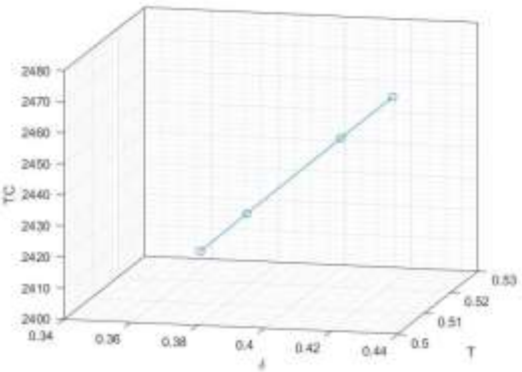


Fig.10. Effect of Change M on Total Cost

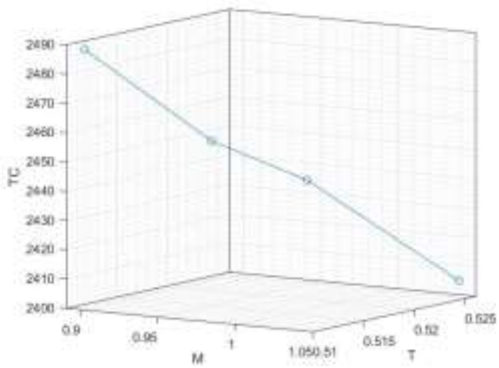
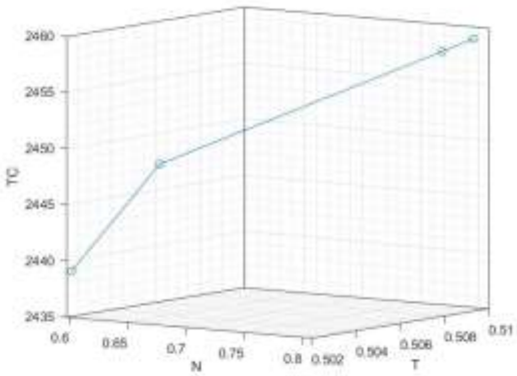


Fig.11. Effect of Change N on Total Cost



4. SENSITIVITY ANALYSIS:

Now, let us study on changes in the values of the system parameters based on the optimal replenishment policy of Example 3. One parameter is changed at a time, keeping the other parameters unchanged. The results are summarized in Table 1. Based on the numerical results, we obtain the following managerial implications

1. The total cost (TC) is highly sensitive with the change of the parameters a, b and K. The retailer should give more concentration in selecting these parameters for taking the optimal decision. It is moderately sensitive with the change in the parameters δ, M and N and it is less sensitive with change in the parameter θ.
2. The order quantity (Q) is reasonably sensitive with the change of the parameters a, b, K and it is less sensitive for the parameters θ, δ, M, N.
3. From Table, we see that the increases in rate of deterioration θ leads to increase in the total cost. Hence when the deterioration rate of products is more, the retailer should order less.
4. The backlogging rate decreases with increase in the backlogging parameter δ. Hence, when the backlogging rate decreases, the total cost increases. To achieve minimum total cost , the retailer should increases the backlogging rate by ordering more quantity.
5. As the length of credit period M increases, the total optimal cost per unit time decreases. This clearly suggests that if the permissible delay period increases, then it helps the retailer to prolong the payment to the supplier without penalty and earn more from the interest earned and eventually results in minimum cost. To acquire minimum cost, the retailer have to concentrate on their credit period tactics and cycle length.
6. If the length of the period N increases, the total cost increases slightly. Due to the increase in the trade credit period of the customer, measure of total cost for the retailer is slightly higher

5. CONCLUSION:

This article provides an EOQ model for imperfect quality deteriorating items with stock dependent demand under two level trade credit period. The two-level trade credit policy is adopted which means the supplier offers the retailer a trade credit period and the retailer also offers credit period to the customer, which will lure more customers. Shortages are allowed and partially backlogged. Backlogging rate is inversely proportional to the waiting time of the next replenishment. After formulating the model solution procedures have been developed to determine the optimal cycle length for the retailer. Behaviour of different parameters have been discussed through the numerical example and sensitivity analysis. Sensitivity analysis are presented to show the relationship between the changing parameter and the decision variables. From sensitivity analysis carried out the rate of change in the parameters K, a, b, θ, M, N and δ is analyzed which helps to the business organization to make better managerial decision. From the results obtained, we see that the retailer can reduce total inventory cost by 1) offering permissible delay in payment to the customer which is received from the supplier to lure more customers 2) ordering less quantity when the supplier provides a permissible delay in payments for stock dependent imperfect quality deteriorating items and learning effect to reduce the percentage of defective items in each replenishment.

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