

Study of Sequencing Problem using different ranking techniques in Fuzzy Environment

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ARTICLE INFO	ABSTRACT
	<p>In this paper, comparison between Yager's ranking and Robust ranking is done on job sequencing problems, in which processing times are taken as trapezoidal fuzzy number. Firstly sequencing problem is solved by using proposed algorithm and then Yager's ranking and robust ranking is applied for the defuzzification of fuzzy job sequencing problems. With the help of numerical example it is shown that Yager's ranking is more effective for handling fuzzy sequencing problem than robust ranking.</p> <p>Key-Words: Trapezoidal fuzzy number, crisp number, sequencing problem, optimal sequence</p>

1. Introduction

Job sequencing is an important branch of operation research in which order of jobs on number of machines is selected to minimize total elapsed time. In sequencing problems systematic procedure is followed to find the sequence of jobs. Processing times taken in job sequencing problems are in the form of crisp numbers that might not be accurate and imprecise. To overcome the problem of Vagueness Zadeh [17] introduced the concept of fuzzy numbers. Dubois and Prade [2] proposed fuzzy numbers as a fuzzy subset of the real line in which membership values clustered around a real number called mean value. Yager[15] defined ranking functions. Cahon[9,10] used fuzzy in inventory control and developed fuzzy job sequencing for a job shop. Ishii and Tada[4] solved single machine scheduling with fuzzy precedence relation. Ishubuchi and Lee[5] also worked in scheduling under fuzzy environment. Yuan et al.[16] solved scheduling problem with fuzzy due dates. Jadhav and Bajaj[6] used average high ranking method for defuzzification in scheduling problems. Kripa and Govindarajan [8] solved job sequencing problem by trapezoidal fuzzy numbers. Gupta et al.[3] proposed new method for obtaining optimal sequence and compared with Johnson's method. Sahoo [11] used Yagers' ranking index method for defuzzification of fuzzy numbers. Selvakumari and Santhi [12,13] worked with pentagonal and octagonal fuzzy numbers and Selvakumari and Sowmiya [14] used hexagonal fuzzy number for solving Sequencing problem. Kanchana et al.[7] has taken processing times as triangular fuzzy numbers and by using ranking solved job sequencing problems.

2. Preliminaries

Definition 2.1

A crisp set or a classical set A is defined as a collection of distinct and distinguishable objects. The objects are called elements of A .

Definition 2.2

A crisp set A , defined on the universal set X , can also be represented by $A = \{(x, \mu_A(x)); x \in X\}$ where $\mu_A : X \rightarrow \{0,1\}$ is called characteristic function defined by

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

Definition 2.3

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X . This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range $[0,1]$ i.e $\mu_{\tilde{A}}: X \rightarrow [0,1]$. The assigned values indicate the membership grade of the element in the set A . The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)): x \in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition 2.4

A fuzzy set \tilde{A} , defined on the universal set of real numbers R , is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}}: R \rightarrow [0,1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, c] \cup [d, \infty)$.
3. Is strictly increasing on $[c, a]$ and strictly decreasing on $[b, d]$.
4. $\mu_{\tilde{A}}(x) = 1$ for all $x \in [a, b]$.

Definition 2.5

A fuzzy number $\tilde{A} = (a, b, c, d)$, is said to be a trapezoidal fuzzy number, if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{(d-x)}{(d-c)}, & c \leq x \leq d \\ 0, & \text{otherwise} \end{cases}$$

where $a, b, c, d \in R$

Definition 2.6(Yager's ranking)

The ranking function of trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is defined as

$$R(\tilde{A}) = \frac{2a+2b+d-c}{4}.$$

Definition 2.7(Robust ranking)

A robust ranking technique satisfies costs, linearity and additive properties. Assuming a convex fuzzy number, the robust ranking index is expressed as $R(\alpha)$, where α is the α -level cut of the fuzzy number.

3. Algorithm to solve fuzzy job sequencing problems[8]

We have many types of sequencing problems. In every problem objective is to minimize the total elapsed time and waiting time for the jobs. Here, let us consider the problem consisting of n-jobs processed through 2-machines. In this method, processing time of machines is taken as trapezoidal fuzzy numbers and thus sequencing problem can be written as follows:

Jobs→ Machines↓	J_1	J_2	J_n
M_1	\tilde{t}_{11}	\tilde{t}_{12}	\tilde{t}_{1n}
M_2	\tilde{t}_{21}	\tilde{t}_{22}	\tilde{t}_{2n}

Here \tilde{t}_{ij} is trapezoidal fuzzy numbers which denotes time duration taken by i^{th} job on j^{th} machine.

Stepwise algorithm is as follows:

Step1. Start

Step 2. Find ranking function of all processing times t_{ij} .

Step 3. Add processing times of two machines corresponding to each job.

Step 4. From these numbers, find the smallest sum and if it corresponds to first machine then place that job in the first available position in the sequence.

Step 5. If this is for second machine, then write that job in the last available position in the sequence. In case of tie : if it is for same machine then select lowest subscript job for machine 1 and largest subscript for machine 2.

Step 6. Do not consider jobs that have assigned position.

Step 7. Repeat these steps until all the jobs are included in the sequence.

Step 8. Calculate overall elapsed time and idle time for both machines.

Step 9. End.

3.1 Numerical Illustration

Example[1]

There are five tasks A, B, C, D and E which must go through two machines M_1 and M_2 . The fuzzy processing times for all the tasks on two machines are given below:

Jobs→ Machines↓	A	B	C	D	E
M_1	(4,4.5,5.5,6)	(0,0.5,1.5,2)	(8,8.5,9.5,10)	(2,2.5,3.5,4)	(9,9.5,10.5,11)
M_2	(1,1.5,2.5,3)	(5,5.5,6.5,7)	(6,6.5,7.5,8)	(7,7.5,8.5,9)	(3,3.5,4.5,5)

Solution:

On solving this question by using Robust Ranking[1] and an algorithm explained in section 3 the optimal sequence obtained is

B	D	C	E	A
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and total elapsed time obtained is (29,29.5,30.5,31) \approx 30 hours.

Idle time for machine is M_1 (1,1.5,2.5,3) \approx 2 hours

Idle time for machine is M_2 (2,2.5,3.5,4) \approx 3 hours

Further, on solving this question by using Yager's Ranking[15] the algorithm proposed or explained in section 3, the optimal sequence obtained is

B	D	C	E	A
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The minimum total elapsed time is given by:

Job Sequence	M_1		M_2	
	Time in	Time out	Time in	Time out
B	0	0.25	0.25	3.00
D	0.25	1.25	3.00	6.75
C	1.25	5.5	6.75	10.00
E	5.5	10.25	10.25	12.00
A	10.25	14.625	14.625	15.875

Thus the minimum total elapsed time \approx 15.875 hours.

Idle time for machine is $M_1 \approx$ 1.25 hours.

Idle time for machine is $M_2 \approx$ 0.25 hours.

4. Comparison between both rankings:

In this section comparison between Yager's ranking and Robust ranking is explained with the help of a table 4.1

	Robust Ranking	Yager's Ranking
Total elapsed Time	30 hours	15.875 hours
Idle time for machine M_1	2 hours	1.25 hours
Idle time for machine M_2	3 hours	0.25 hours

Table 4.1

Job sequencing problem is solved with two above said rankings and table 4.1 shows that if we use Yager's ranking then total elapsed time comes out is much less as compared to solution done with Robust ranking.

Conclusion:

In this paper job sequencing problem is taken in which processing times are taken as trapezoidal fuzzy numbers. Using Robust ranking and Yager's ranking problem is solved and comparison is done between two which shows that Yager's ranking provides better result than Robust ranking.

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