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New Form Of Continuous Functions In Topological Spaces

Basavaraj M. Ittanagi[1*] and Prakash R[2]

[1*]Department of Mathematics, Siddaganga Institute of Technology, Autonomous Institution Affiliated to VTU, Tumakuru-572103, KarnatakaState, India. E-mail: dr.basavarajsit@gmail.com
[2]Department of Mathematics, RV College of Engineering, Autonomous Institution Affiliated to VTU, Mysore Road, Bengaluru–560 059, KarnatakaState, India. E-mail: prakashr@rvce.edu.in

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ABSTRACT

In this paper a new class of functions called semi maximal continuous, semi maximal irresolute, semi maximal semi continuous, super maximal semi continuous and super semi maximal continuous functions are introduced and investigated. A function $f: X \to Y$ is said to semi maximal continuous if for every maximal open set M in Y, f^{-1} (M) is semi maximal open set in X. During this process, some of their properties are obtained.

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Key words and phrases: Minimal open sets, maximal open sets, semi - maximal open and semi - minimal closed sets.

1. Introduction and Preliminaries

In the year 1963, N. Levine [5] introduced and studied semi-open sets. The family of all semi-open sets of X is denoted by SO(X). The complement of semi-open set is called semi-closed set [4] of X. The family of all semi - closed sets of X is denoted by SC(X).
In the years 2001 and 2003, F. Nakaoka and N. Oda [6] and [7], introduced and studied minimal open (resp. minimal closed) sets and maximal open (resp. maximal closed) sets in topological spaces respectively. The family of all minimal open (resp. minimal closed) sets in a topological space X is denoted by $M_iO(X)$ (resp. $M_iC(X)$) and clearly all minimal open (resp. minimal closed) sets are open (resp. closed) sets. Also the family of all maximal open (resp. maximal closed) sets in a topological space X is denoted by $M_aO(X)$ (resp. $M_aC(X)$) and clearly all maximal open (resp. maximal closed) sets are open (resp. closed) sets. It is observed that every minimal open (resp. minimal closed) sets and maximal open (resp. maximal closed) sets are independent of each other. Therefore M is a minimal open (resp. maximal open) set if and only if X − M is maximal closed (resp. minimal closed) set.
In the year 2009, S. S. Benchalli and Basavaraj M. Ittanagi [1], introduced and studied semi - maximal open and semi - minimal closed sets in topological spaces. The family of all semi-maximal open sets in a topological space X is denoted by $SM_aO(X)$. It is observed that every maximal open set is semi - maximal open set but not the converse. The family of all semi - maximal closed sets in a topological space X is denoted by $SM_aC(X)$. It is observed that every minimal closed set is semi - minimal closed set but not the converse.
In the year 2011, S. S. Benchalli and Basavaraj M. Ittanagi [2], introduced and studied minimal continuous, maximal continuous, minimal irresolute, maximal irresolute, minimal- maximal continuous and maximal-minimal continuous maps in topological spaces. Also in 2018, Basavaraj M. Ittanagi and G. P. Siddapur [3], introduced and investigated maximal semi continuous, strongly maximal semi continuous, maximal semi irresolute, maximal semi s-continuous and super maximal semi continuous functions in topological spaces.
1.1 Definition [5]: A subset A of a topological space X is said to be semi open set if there exists some open set U such that $U \subset A \subset cl\,(U)$.
1.2 Definition [6]: A proper nonempty open subset U of a topological space X is said to be a minimal open set if any open set which is contained in U is ϕ or U.
1.3 Definition [6]: A proper nonempty open subset U of a topological space X is said to be maximal open set if any open set which contains U is X or U.
1.4 Definition [7]: A proper nonempty closed subset F of a topological space X is said to be a minimal closed set if any closed set which is contained in F is ϕ or F.

1.5 Definition [7]: A proper nonempty closed subset F of a topological space X is said to be maximal closed set if any closed set which contains F is X or F.
1.6 Definition [1]: A proper nonempty semi open subset M of a topological space X is said to be maximal semi open set if any semi open set which contains M is X or M itself.
1.7 Definition [1]: A proper nonempty semi closed subset N of a topological space X is said to be a minimal semi closed set if any closed set which is contained in N is ϕ or F itself.
1.8 Definition [1]: A set A in a topological space X is said to be semi maximal open set if there exists a maximal open set M such that $M \subset A \subset cl\,(M)$.
1.9 Definition [1]: A subset N of a topological space X is said to be semi minimal closed if $(X - N)$ is semi maximal open.
1.10 Definition [5]: Let X and Y be the topological spaces. A map f: X→Y is said to be semi continuous if $f^{-1}(U)$ is a semi open set in X for every open set U in Y.
1.11 Definition [2]: Let X and Y be the topological spaces. A map f: X→Y is called
i) minimal continuous if $f^{-1}(M)$ is an open set in X for every minimal open set M in Y.
ii) maximal continuous if $f^{-1}(M)$ is an open set in X for every maximal open set M in Y.
iii) minimal irresolute if $f^{-1}(M)$ is minimal open set in X for every minimal open set M in Y.
iv) maximal irresolute if $f^{-1}(M)$ is maximal open set in X for every maximal open set M in Y.
v) minimal-maximal continuous if $f^{-1}(M)$ is maximal open set in X for every minimal open set M in Y.
vi) maximal-minimal continuous if $f^{-1}(M)$ is minimal open set in X for every maximal open set M in Y.
1.12 Definition [3]: Let X and Y be topological spaces. A function f: X → Y is said to be i) maximal semi continuous if for every maximal open set M in Y, $f^{-1}(M)$ is semi open set in X.
ii) strongly maximal semi continuous if for every maximal open set M in Y, $f^{-1}(M)$ is maximal semi open set in X.
iii) maximal semi irresolute if for every maximal semi open set M in Y, $f^{-1}(M)$ is maximal semi open set in X.
iv) maximal semi s-continuous if for every maximal semi open set M in Y, $f^{-1}(M)$ is semi open set in X.
v) super maximal semi continuous if for every maximal semi open set M in Y, $f^{-1}(M)$ is semi maximal open set in X.

2. Semi Maximal Continuous Functions.

2.1 Definition: Let X and Y be topological spaces. A function $f : X \to Y$ is said to be semi maximal continuous (briefly Semi Max. cont.) if for every maximal open set M in Y, $f^{-1}(M)$ is semi maximal open set in X.
2.2 Theorem: Every strongly maximal continuous function is semi maximal continuous.
Proof: Let $f : X \to Y$ be a strongly maximal continuous function. To prove f is semi maximal continuous. Let M be any maximal open set in Y. Since f is strongly maximal continuous, we have $f^{-1}(M)$ is a maximal open set in X. Since every maximal open set is a semi maximal open set, we have $f^{-1}(M)$ is a semi maximal open set in X. Thus, $f^{-1}(M)$ is a semi maximal open set in X, for every maximal open set M in Y. Hence f is semi maximal continuous.
2.3 Remark: The converse of the above Theorem 2.2 need not be true.
2.4 Example: Let X = Y = {a, b, c, d} with topologies $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\mu = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}, Y\}$. Let $f : (X, \tau) \to (Y, \mu)$ be an identity function. Then f is semi maximal continuous but not strongly maximal continuous. Now $\{a, b, c\} \in MO(Y)$, then $f^{-1}(\{a, b, c\}) = \{a, b, c\} \notin MO(X)$.
2.5 Theorem: Every semi maximal continuous function is maximal semi continuous.
Proof: Let $f : X \to Y$ be a semi maximal continuous function. To prove f is maximal semi continuous. Let M be any maximal open set in Y. Since f is semi maximal continuous, we have $f^{-1}(M)$ is a semi maximal open set in X. Since every semi maximal open set is a semi open set, we have $f^{-1}(M)$ is a semi open set in X. Thus, $f^{-1}(M)$ is a semi open set in X, for every maximal open set M in Y. Hence f is maximal semi continuous.
2.6 Remark: The converse of the above Theorem 2.5 need not be true.
2.7 Example: Let X = Y = {a, b, c} with topologies $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\mu = \{\phi, \{a\}, \{a, c\}, Y\}$. Let $f : (X, \tau) \to (Y, \mu)$ be an identity function. Then f is maximal semi continuous but not semi maximal continuous. Now $\{a, c\} \in MO(Y)$, then $f^{-1}(\{a, c\}) = \{a, c\} \notin SMO(X)$.
2.8 Theorem: Let X and Y be topological spaces. A mapping f: X → Y is semi maximal continuous if and only if the inverse image of each minimal closed set in Y is semi minimal closed set in X.
Proof: Assume that f is a semi maximal continuous and let F be any minimal closed set in Y, Y – F is maximal open set in Y. To show that $f^{-1}(F)$ is a semi minimal closed set in X. Since f is semi maximal continuous, we have $f^{-1}(Y - F)$ is a semi maximal open set in X. But $f^{-1}(Y - F) = X - f^{-1}(F)$ is a semi maximal open set in X. Therefore $f^{-1}(F)$ is a semi minimal closed set in X.
Conversely, let $f^{-1}(F)$ be a semi minimal closed set in X for every minimal closed set in Y. We want show that f is semi maximal continuous. Let M be any maximal open set in Y. Then Y – M is minimal closed set in Y

and so by hypothesis $f^{-1}(Y - M) = X - f^{-1}(M)$ is a semi minimal closed set in X, that is $f^{-1}(M)$ is a semi maximal open set in X. Hence f is semi maximal continuous.

2.9 Theorem: Let X and Y be topological spaces and A be a nonempty subset of X. If $f : X \to Y$ is semi maximal continuous then restriction function $f_A : A \to Y$ is semi maximal continuous, where A has the relatived topology.

Proof: Let $f : X \to Y$ be a semi maximal continuous function and let A be a nonempty subset of X. Let M be a maximal open set in Y. Since f is semi maximal continuous, we have $f^{-1}(M)$ is a semi maximal open set in X. Therefore by definition of $f_A : A \to Y$ it is evident that $f_A^{-1}(M) = A \cap f^{-1}(M)$. Therefore by definition of relative topology $A \cap f^{-1}(M)$ is a semi maximal open set in A. Therefore by definition $f_A : A \to Y$ is semi maximal continuous.

2.10 Theorem: Let X and Y be topological spaces. Then $f : X \to Y$ is semi maximal continuous if and only if for any point $p \in X$ and for any maximal open set M in Y containing $f(p)$, then there exists a semi maximal open set N in X such that $p \in N$ and $f(N) \subset M$.

Proof: Let M be any maximal open set in Y containing $f(p)$ for every point $p \in N$, where N is semi maximal open set in X. Since f is semi maximal continuous, we have $f^{-1}(M)$ is a semi maximal open set in X. Take $N = f^{-1}(M)$ which implies $f(N) = f(f^{-1}(M)) \subset M$. Therefore $f(N) \subset M$.

Conversely, let M be any maximal open set in Y. To prove $f^{-1}(M)$ is a semi maximal open set in X. Therefore by hypothesis, there exists a semi maximal open set N in X, such that $p \in N$ which implies $f(p) \in f(N) \subset M$ which implies $p \in f^{-1}(f(N)) \subset f^{-1}(M)$. Thus $f^{-1}(M)$ is a semi maximal open set in X, for every maximal open set M in Y. Therefore f is semi maximal continuous.

2.11 Remark: The composition of semi maximal continuous need not be semi maximal continuous, which is shown by the following example.

2.12 Example: Let X = Y = Z = {a, b, c, d} with topologies $\tau = \{\phi, \{a, b\}, \{c, d\}, X\}$, $\mu = \{\phi, \{a\}, \{a, b\}, Y\}$, and $\eta = \{\phi, \{a\}, \{a, b\}, \{a, b, d\}, Z\}$. Let $f : (X, \tau) \to (Y, \mu)$ and $g : (Y, \mu) \to (Z, \eta)$ are identity functions. Then f and g are semi maximal continuous functions but not $g \circ f : (X, \tau) \to (Z, \eta)$ is semi maximal continuous. Now $\{a, b, d\} \in MO(Z)$, then $(g \circ f)^{-1}(\{a, b, d\}) = \{a, b, d\} \notin SMO(X)$.

2.13 Theorem: If $f : X \to Y$ is strongly maximal continuous (resp. semi maximal continuous) and $g : Y \to Z$ is strongly maximal continuous functions. Then $g \circ f : X \to Z$ is semi maximal continuous.

3. Semi Maximal Irresolute

3.1 Definition: Let X and Y be topological spaces. A function $f : X \to Y$ is said to be semi maximal irresolute (briefly Semi Max. irre.) if for every semi maximal open set M in Y, $f^{-1}(M)$ is semi maximal open set in X.

3.2 Theorem: Every semi maximal irresolute is semi maximal continuous.

Proof: Let $f : X \to Y$ be a semi maximal irresolute. To prove f is semi maximal continuous. Let M be any maximal open set in Y. Since every maximal open set is a semi maximal open set, we have M is a semi maximal open set in Y. Since f is semi maximal irresolute, we have $f^{-1}(M)$ is a semi maximal open set in X. Thus, $f^{-1}(M)$ is a semi maximal open set in X, for every maximal open set M in Y. Hence f is semi maximal continuous.

3.3 Remark: The converse of the above Theorem 3.2 need not be true.

3.4 Example: Let X = Y = {a, b, c, d} with topologies $\tau = \{\phi, \{a, b\}, \{c, d\}, X\}$ and $\mu = \{\phi, \{a\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \mu)$ be an identity function. Then f is semi maximal continuous but not semi maximal irresolute. Now $\{a, b, d\} \in SMO(Y)$, then $f^{-1}(\{a, b, d\}) = \{a, b, d\} \notin SMO(X)$.

3.5 Theorem: Every semi maximal irresolute is maximal semi continuous.

Proof: Let $f : X \to Y$ be a semi maximal irresolute. To prove f is maximal semi continuous. Let M be any maximal open set in Y. Since every maximal open set is a semi maximal open set, we have M is a semi maximal open set in Y. Since f is semi maximal irresolute, we have $f^{-1}(M)$ is a semi maximal open set in X. Since every semi maximal open set is semi open set, we have $f^{-1}(M)$ is a semi open set in X. Thus, $f^{-1}(M)$ is a semi open set in X, for every maximal open set M in Y. Hence f is maximal semi continuous.

3.6 Remark: The converse of the above Theorem 3.5 need not be true.

3.7 Example: By Example 3.4, we have f is maximal semi continuous but not semi maximal irresolute. Now $\{a, b, d\} \in SMO(Y)$, then $f^{-1}(\{a, b, d) = \{a, b, d\} \notin SO(X)$.

3.8 Theorem: Let X and Y be topological spaces. A mapping $f : X \to Y$ is semi maximal irresolute if and only if the inverse image of each semi minimal closed set in Y is semi minimal closed set in X.

Proof: Assume that f is a semi maximal irresolute and let F be any semi minimal closed set in Y, then $Y - F$ is semi maximal open set in Y. To show that $f^{-1}(F)$ is a semi minimal closed set in X. Since f is semi maximal irresolute, we have $f^{-1}(Y - F)$ is a semi maximal open set in X. But $f^{-1}(Y - F) = X - f^{-1}(F)$ is a semi maximal open set in X. Therefore $f^{-1}(F)$ is a semi minimal closed set in X.

Conversely, let f^{-1}(F) be a semi minimal closed set in X for every semi minimal closed set in Y. To show that f is semi maximal irresolute. Let M be any semi maximal open set in Y. Then Y–M is a semi minimal closed set in Y and so by hypothesis $f^{-1}(Y - M) = X - f^{-1}(M)$ is a semi minimal closed set in X, that is $f^{-1}(M)$ is a semi maximal open set in X. Hence f is semi maximal irresolute.

3.9 Theorem: Let X and Y be topological spaces and A be a nonempty subset of X. If $f : X \to Y$ is semi maximal irresolute then restriction function $f_A : A \to Y$ is semi maximal irresolute, where A has the relatived topology.

Proof: Similar to prove Theorem 2.9.

3.10 Theorem: Let X and Y be topological spaces. Then $f : X \to Y$ is semi maximal irresolute if and only if for any point $p \in X$ and for any semi maximal open set M in Y containing $f(p)$, then there exists a semi maximal open set N in X such that $p \in N$ and $f(N) \subset M$.

Proof: Similar to prove Theorem 2.10.

3.11 Theorem: Let X, Y and Z be topological spaces and $f : X \to Y$ and $g : Y \to Z$ be semi maximal irresolute. Then $g \circ f : X \to Z$ is semi maximal irresolute

4. Semi Maximal Semi Continuous Functions.

4.1 Definition: Let X and Y be topological spaces. A function $f : X \to Y$ is said to be semi maximal semi continuous (briefly Semi Max. semi cont.) if for every semi maximal open set M in Y, $f^{-1}(M)$ is semi open set in X.

4.2 Theorem: Every semi maximal irresolute is semi maximal semi continuous.

Proof: Let $f : X \to Y$ be a semi maximal irresolute. To prove f is semi maximal semi continuous. Let M be any semi maximal open set in Y. Since f is semi maximal irresolute, we have $f^{-1}(M)$ is a semi maximal open set in X. Since every semi maximal open set is a semi open set, we have $f^{-1}(M)$ is a semi open set in X. Thus, $f^{-1}(M)$ is a semi open set in X, for every semi maximal open set M in Y. Hence f is semi maximal semi continuous.

4.3 Remark: The converse of the above Theorem 4.2 need not be true.

4.4 Example: Let X = Y = {a, b, c} with topologies $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\mu = \{\phi, \{a\}, \{a, c\}, Y\}$. Let $f : (X, \tau) \to (Y, \mu)$ be an identity function. Then f is semi maximal semi continuous but not semi maximal irresolute. Now $\{a, c\} \in SMO(Y)$, then $f^{-1}(\{a, c\}) = \{a, c\} \notin SMO(X)$.

4.5 Theorem: Every semi maximal semi continuous function is maximal semi continuous.

Proof: Let $f : X \to Y$ be a semi maximal semi continuous function. To prove f is maximal semi continuous. Let M be any maximal open set in Y. Since every maximal open set is a semi maximal open set, we have M is a semi maximal open set in Y. Since f is semi maximal semi continuous, we have $f^{-1}(M)$ is a semi open set in X. Thus, $f^{-1}(M)$ is a semi open set in X, for every maximal open set M in Y. Hence f is maximal semi continuous.

4.6 Remark: The converse of the above Theorem 4.5 need not be true.

4.7 Example: Let X = Y = {a, b, c, d} with topologies $\tau = \{\phi, \{a, b\}, \{c, d\}, X\}$ and $\mu = \{\phi, \{a\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \mu)$ be an identity function. Then f is maximal semi continuous but not semi maximal semi continuous. Now $\{a, b, c\} \in SMO(Y)$, then $f^{-1}(\{a, b, c\}) = \{a, b, c\} \notin SO(X)$.

4.8 Theorem: Let X and Y be topological spaces. A mapping $f : X \to Y$ is semi maximal semi continuous if and only if the inverse image of each semi minimal closed set in Y is semi closed set in X.

Proof: Assume that f is a semi maximal semi continuous and let F be any semi minimal closed set in Y, Y – F is semi maximal open set in Y. To show that f^{-1}(F) is a semi closed set in X. Since f is semi maximal semi continuous, we have $f^{-1}(Y - F)$ is a semi open set in X. But $f^{-1}(Y - F) = X - f^{-1}(F)$ is a semi open set in X. Therefore f^{-1}(F) is a semi closed set in X.

Conversely, let f^{-1}(F) be a semi closed set in X for every semi minimal closed set in Y. To show that f is semi maximal semi continuous. Let M be any semi maximal open set in Y. Then Y – M is semi minimal closed set in Y and so by hypothesis $f^{-1}(Y - M) = X - f^{-1}(M)$ is a semi closed set in X, that is $f^{-1}(M)$ is a semi open set in X. Hence f is semi maximal semi continuous.

4.9 Theorem : Let X and Y be topological spaces and A be a nonempty subset of X. If $f : X \to Y$ is semi maximal semi continuous then restriction function $f_A : A \to Y$ is semi maximal semi continuous, where A has the relatived topology.

Proof: Similar to prove Theorem 2.9.

4.10 Theorem: Let X and Y be topological spaces. Then $f : X \to Y$ is semi maximal semi continuous if and only if for any point $p \in X$ and for any semi maximal open set M in Y containing $f(p)$, there exists a semi open set N in X such that $p \in N$ and $f(N) \subset M$.

Proof: Similar to prove Theorem 2.10.

4.11 Remark: The composition of semi maximal semi continuous need not be semi maximal semi continuous, which is shown by the following example.

4.12 Example: Let X = Y = Z = {a, b, c} with topologies τ = {ϕ, {b}, {b, c}, X}, μ = {ϕ, {a}, {a, b}, Y}, and η = {ϕ, {a}, {a, c}, Z}. Let f : (X ,τ) → (Y , μ) and g : (Y , μ) → (Z , η) are identity functions. Then f and g are semi maximal semi continuous functions but not g o f : (X, τ) → (Z, η) is semi maximal semi continuous. Now {a, c} ∈ SMO(Z), then (g o f)$^{-1}$ ({a, c})= {a, c} ∉ SO(X).
4.13 Theorem : If f : X → Y is semi maximal irresolute (resp. semi maximal semi continuous) and g : Y → Z is semi maximal continuous functions (resp. semi maximal irresolute). Then g o f : X → Z is semi maximal semi continuous.

5. Super Maximal Semi Continuous Functions.

5.1 Definition: Let X and Y be topological spaces. A function f : X → Y is said to be super maximal semi continuous (briefly Su. Max. semi cont.) if for every maximal semi open set M in Y, f^{-1} (M) is semi maximal open set in X.
5.2 Theorem: Every super maximal semi continuous function is maximal semi continuous.
Proof: Let f : X → Y be a super maximal semi continuous function. To prove f is maximal semi continuous. Let M be any maximal open set in Y. Since every maximal open set is a maximal semi open set, we have M is a maximal semi open set in Y. Since f is super maximal semi continuous, then f^{-1}(M) is a semi maximal open set in X. Since every semi maximal open set is a semi open set, we have f^{-1}(M) is a semi open set in X. Thus, f^{-1} (M) is a semi open set in X, for every maximal open set M in Y. Hence f is maximal semi continuous.
5.3 Remark: The converse of the above theorem 5.2 need not be true.
5.4 Example: Let X = Y = {a, b, c} with topologies τ = {ϕ, {a}, {a, b}, X} and μ = { ϕ, {a}, {a, c}, Y }. Let f : (X ,τ) → (Y, μ) be an identity function. Then f is maximal semi continuous but not super maximal semi continuous. Now {a, c} ∈ MSO(Y), then f^{-1}({a, c}) = {a, c } ∉ SMO(X).
5.5 Theorem: Every super maximal semi continuous function is maximal semi s - continuous.
Proof: Let f : X → Y be a super maximal semi continuous function. To prove f is maximal semi s - continuous. Let M be any maximal semi open set in Y. Since f is super maximal semi continuous, we have f^{-1} (M) is a semi maximal open set in X. Since every semi maximal open set is a semi open set, we have f^{-1} (M) is a semi open set in X. Thus, f^{-1}(M) is a semi open set in X, for every maximal semi open set M in Y. Hence f is maximal semi s - continuous.
5.6 Remark: The converse of the above theorem 5.5 need not be true.
5.7 Example: By example 5.4, we have f is maximal semi s-continuous but not super maximal semi continuous. Now {a, c} ∈ MSO(Y), then f^{-1}({a, c) ={a, c }∉ SMO(X).
5.8 Theorem: Every super maximal semi continuous function is semi maximal continuous.
Proof: Let f : X → Y be a super maximal semi continuous function. To prove f is semi maximal continuous. Let M be any maximal open set in Y. Since every maximal open set is a maximal semi open set, we have M is a maximal semi open set in Y. Since f is super maximal semi continuous, we have f^{-1} (M) is a semi maximal open set in X. Thus, f^{-1}(M) is a semi maximal open set in X, for every maximal open set M in Y. Hence f is semi maximal continuous.
5.9 Remark: The converse of the above theorem 5.8 need not be true.
5.10 Example: Let X = Y = {a, b, c} with topologies τ = {ϕ, {a}, {a, b}, X} and μ = { ϕ, {a}, {a, b}, Y }. Let f : (X ,τ) → (Y, μ) be an identity function. Then f is semi maximal continuous but not super maximal semi continuous. Now {a, c} ∈ MSO(Y), then f^{-1}({a, c}) = {a, c }∉ SMO(X).
5.11 Theorem: Let X and Y be topological spaces. A mapping f : X → Y is super maximal semi continuous if and only if the inverse image of each minimal semi closed set in Y is semi minimal closed set in X.
Proof : Assume that f is a super maximal semi continuous and let F be any minimal semi closed set in Y, Y – F is maximal semi open set in Y. To show that f^{-1}(F) is a semi minimal closed set in X. Since f is super maximal semi continuous, we have f^{-1}(Y – F) is a semi maximal open set in X. But f^{-1}(Y – F) = X – f^{-1}(F) is a semi maximal open set in X. Therefore f^{-1}(F) is a semi minimal closed set in X.
Conversely, let f^{-1}(F) be a semi minimal closed set in X for every minimal semi closed set in Y. We want show that f is super maximal semi continuous. Let M be any maximal semi open set in Y. Then Y – M is minimal semi closed set in Y and so by hypothesis f^{-1}(Y – M) = X – f^{-1}(M) is a semi minimal closed set in X, that is f^{-1}(M) is a semi maximal open set in X. Hence f is super maximal semi continuous.
5.12 Theorem: Let X and Y be topological spaces and A be a nonempty subset of X. If f : X → Y is super maximal semi continuous then restriction function f_A : A → Y is super maximal semi continuous, where A has the relatived topology.
Proof: Similar to prove Theorem 2.9.

5.13 Theorem: Let X and Y be topological spaces. Then $f : X \to Y$ is super maximal semi continuous if and only if for any point $p \in X$ and for any maximal semi open set M in Y containing $f(p)$, there exists a semi maximal open set N in X such that $p \in N$ and $f(N) \subset M$.
Proof: Similar to prove Theorem 2.10.
5.14 Theorem : If $f : X \to Y$ is super maximal semi continuous and $g : Y \to Z$ is maximal semi irresolute. Then $g \circ f : X \to Z$ is super maximal semi continuous.
5.15 Theorem : If $f : X \to Y$ is super maximal semi continuous and $g : Y \to Z$ is strongly maximal semi continuous. Then $g \circ f : X \to Z$ is super maximal semi continuous.

6. Super Semi Maximal Continuous Functions.

6.1 Definition: Let X and Y be topological spaces. A function $f : X \to Y$ is said to be super semi maximal continuous (briefly Su. Max. semi cont.) if for every semi maximal open set M in Y, $f^{-1}(M)$ is maximal semi open set in X.
6.2 Theorem: Every super semi maximal continuous function is maximal semi continuous.
Proof: Let $f : X \to Y$ be a super semi maximal continuous function. To prove f is maximal semi continuous. Let M be any maximal open set in Y. Since every maximal open set is a semi maximal open set, we have M is semi maximal open set in Y. Since f is super semi maximal continuous, we have $f^{-1}(M)$ is a maximal semi open set in X. Since every maximal semi open set is semi open set, we have $f^{-1}(M)$ is a semi open set in X. Thus, $f^{-1}(M)$ is a semi open set in X, for every maximal open set M in Y. Hence f is maximal semi continuous.
6.3 Remark: The converse of the above theorem 6.2 need not be true.
6.4 Example: Let X = Y = {a, b, c, d} with topologies $\tau = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}, X\}$ and $\mu = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \mu)$ be an identity function. Then f is maximal semi continuous but not super semi maximal continuous. Now $\{a, b\} \in SMO(Y)$, then $f^{-1}(\{a, b\}) = \{a, b\} \notin MSO(X)$.
6.5 Theorem: Every super semi maximal continuous function is strongly maximal semi continuous.
Proof: Let $f : X \to Y$ be a super semi maximal continuous function. To prove f is strongly maximal semi continuous. Let M be any maximal open set in Y. Since every maximal open set is a semi maximal open set, we have M is a semi maximal open set in Y. Since f is super semi maximal continuous, we have $f^{-1}(M)$ is a maximal semi open set in X. Thus, $f^{-1}(M)$ is a maximal semi open set in X, for every maximal open set M in Y. Hence f is strongly maximal semi continuous.
6.6 Remark: The converse of the above theorem 6.5 need not be true.
6.7 Example: By Eexample 6.4, f is strongly maximal semi continuous but not super semi maximal continuous. Now $\{a, b, c\} \in SMO(Y)$, then $f^{-1}(\{a, b, c) = \{a, b, c\} \notin MSO(X)$.
6.8 Theorem: Every super semi maximal continuous function is semi maximal semi continuous.
Proof: Let $f : X \to Y$ be a super semi maximal continuous function. To prove f is semi maximal semi continuous. Let M be any semi maximal open set in Y. Since f is super semi maximal continuous, we have $f^{-1}(M)$ is a maximal semi open set in X. Since every maximal semi open set is a semi open set, we have $f^{-1}(M)$ is a semi open set in X. Thus, $f^{-1}(M)$ is a semi open set in X, for every semi maximal open set M in Y. Hence f is semi maximal semi continuous.
6.9 Remark: The converse of the above theorem 6.8 need not be true.
6.10 Example: By Example 6.4, f is semi maximal semi continuous but not super semi maximal continuous. Now $\{a, b\} \in SMO(Y)$, then $f^{-1}(\{a, b\}) = \{a, b\} \notin MSO(X)$.
6.11 Theorem: Let X and Y be topological spaces. A mapping $f : X \to Y$ is super semi maximal continuous if and only if the inverse image of each semi minimal closed set in Y is minimal semi closed set in X.
Proof : Assume that f is a super semi maximal continuous function and let F be any semi minimal closed set in Y, Y – F is semi maximal open set in Y. To show that $f^{-1}(F)$ is a minimal semi closed set in X. Since f is super semi maximal continuous, we have $f^{-1}(Y - F)$ is a maximal semi open set in X. But $f^{-1}(Y - F) = X - f^{-1}(F)$ is a maximal semi open set in X. Therefore $f^{-1}(F)$ is a minimal semi closed set in X.
Conversely, let $f^{-1}(F)$ be a minimal semi closed set in X for every semi minimal closed set in Y. We want show that f is super semi maximal continuous. Let M be any semi maximal open set in Y. Then Y – M is semi minimal closed set in Y and so by hypothesis $f^{-1}(Y - M) = X - f^{-1}(M)$ is a minimal semi closed set in X, that is $f^{-1}(M)$ is a maximal semi open set in X. Hence f is super semi maximal continuous.
6.12 Theorem: Let X and Y be topological spaces and A be a nonempty subset of X. If $f : X \to Y$ is super semi maximal continuous then restriction function $f_A : A \to Y$ is super semi maximal continuous, where A has the relatived topology.

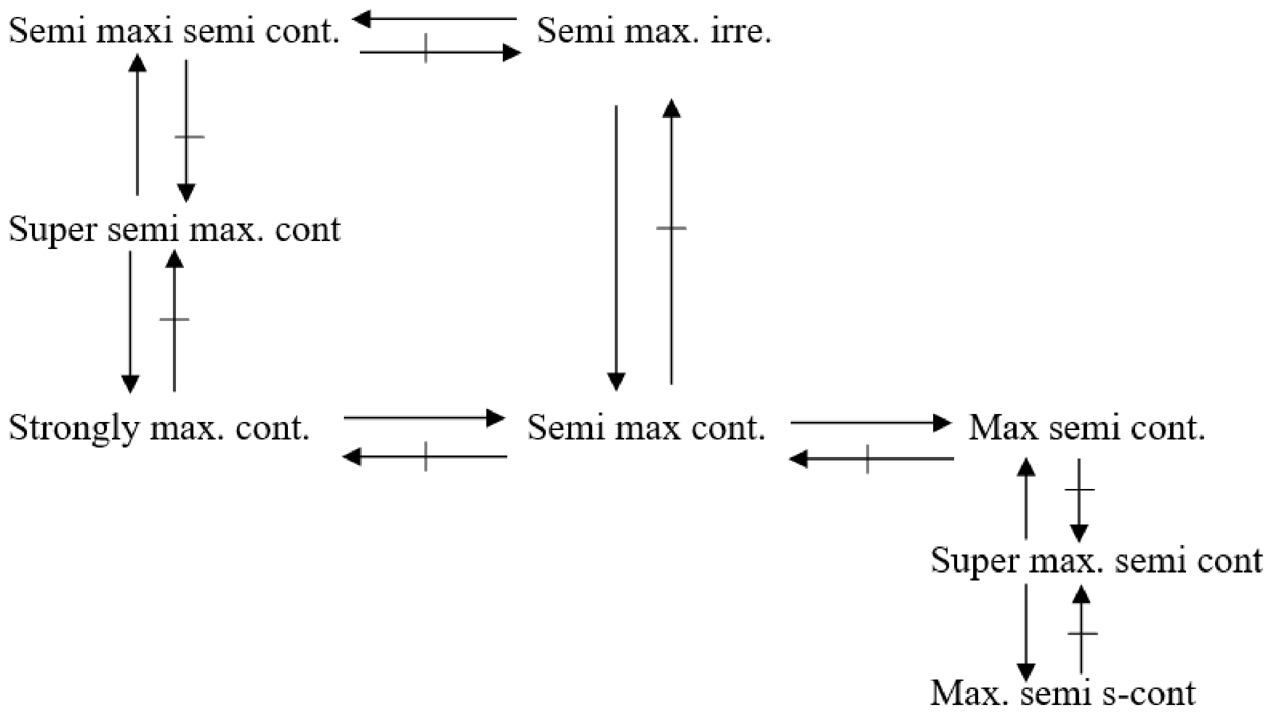
6.13 Theorem: Let X and Y be topological spaces. Then f : $X \to Y$ is super semi maximal continuous if and only if for any point $p \in X$ and for any semi maximal open set M in Y containing $f(p)$, there exists a maximal semi open set N in X such that $p \in N$ and $f(N) \subset M$.

6.14 Remark: The composition of super semi maximal continuous need not be super semi maximal continuous, which is shown by the following example.

6.15 Example: Let X = Y = Z = {a, b, c} with topologies $\tau = \{\phi, \{b\}, \{b, c\}, X\}$, $\mu = \{\phi, \{a\}, \{a, b\}, Y\}$, and $\eta = \{\phi, \{c\}, \{a, c\}, Z\}$. Let $f : (X, \tau) \to (Y, \mu)$ and $g : (Y, \mu) \to (Z, \eta)$ are identity functions. Then f and g are super semi maximal continuous functions but not $g \circ f : (X, \tau) \to (Z, \eta)$ is super semi maximal continuous. Now $\{a, c\} \in SMO(Z)$, then $(g \circ f)^{-1}\{a, c\} = \{a, c\} \notin MSO(X)$.

6.16 Theorem : If $f : X \to Y$ is super semi maximal continuous and $g : Y \to Z$ is semi maximal irresolute. Then $g \circ f : X \to Z$ is super semi maximal continuous.

6.17 Remark: Let $f : X \to Y$ be a function. Then we get the following diagram.



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