



Some Properties Of Sequences In Fuzzy Soft Sequential Sets

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ABSTRACT

Focus of this paper is to analyze characteristics of sequences in fuzzy soft sequential sets. We impose the concept of sequences like bounded sequence, increasing sequence, decreasing sequence, convergent sequence, subsequence in fuzzy soft sequential sets and investigate some properties of the above concepts are also explore.

Keywords: fuzzy soft set, fuzzy soft sequential set, bounded fuzzy soft sequential set.

1. Introduction

The concept of fuzzy sets was introduced by **L.A.Zadeh** [11] in 1965, which is dealing with uncertainty and representing vague concepts. In 1999, **Molodtsov** [7] introduced the concept of soft set theory, which provides a new mathematical theory for dealing with uncertainty. In continuation, **Maji et al** [6] defined and studied the theory of soft sets which is used to construct new soft sets from the given soft sets in the year 2003. In recent years, the researchers have contributed a lot towards fuzzification of soft set theory. In 2001, **Maji et al** [5] proposed the concept of fuzzy soft set which is a new mathematical approach to vagueness by involving the ideas of both fuzzy sets and soft sets. Fuzzy soft sets was further revised and improved by **Ahmad and Kharal** [1] in the year 2009. In 2011, **Tanay and Kandemir** [10] introduced the topological structure of fuzzy soft sets and studied some of its structural properties. In 2002, **Bose and Indrajit Lahiri** [3] introduced the concept of sequential topological spaces. He defined any sequence of subsets of a non void set is called a sequential set. In this paper, we propose characteristics of sequences in fuzzy soft sequential set and analyze its properties.

2. Preliminaries

Definition 2.1 [11] A fuzzy set X over a universal set U is a set defined by a function μ_x performing a mapping $\mu_x : U \rightarrow [0,1]$, here this μ_x is the membership function of X , and the value $\mu_x(u)$ will be the grade of membership of $u \in U$.

Definition 2.2 [7] Let U be an initial universe set and E be a set of parameters. A pair (F, E) is called a soft set over U if and only if F is a mapping of E into the set of all subsets of the set U .

Definition 2.3 [5] Let U be a common universe, E be a set of parameters and $A \subseteq E$. A pair (F, A) is called a fuzzy soft set over U , where F is a mapping given by $F: A \rightarrow \mu_x(U)$, where $\mu_x(U)$ denote the set of all fuzzy subsets of U .

Definition 2.4 [10] Let U be a common universe, E be a set of parameters and $A, B \subseteq E$. Let $(F, A), (G, B)$ be an element of $FS(U, E)$, (Briefly, family of all fuzzy soft subsets) and $\tilde{\tau}$ be a subfamily of $FS(U, E)$. Then $\tilde{\tau}$ is called a fuzzy soft topology on U if the following conditions are satisfied:

- i. $\tilde{\emptyset}, U \in \tilde{\tau}$
- ii. $(F, A), (G, B) \in \tilde{\tau} \Rightarrow (F, A) \tilde{\cap} (G, B) \in \tilde{\tau}$
- iii. $\{(F, A)_k | k \in K\} \subset \tilde{\tau} \Rightarrow \bigcup_{k \in K} (F, A)_k \in \tilde{\tau}$.

The pair (U, τ) is called a fuzzy soft topological space.

Definition 2.5 [11] A is contained in B (or, equivalently, A is a subset of B , or A is smaller than or equal to B) if and only if $\mu_A \leq \mu_B$. In symbols, $A \subset B \Leftrightarrow \mu_A \leq \mu_B$.

Definition 2.6 [3] Any sequence of subsets of a non void set X is called a sequential set in X . That is, $A(s) = \{A_n\}_{n=1}^\infty$, where each A_n is a subset of X , is a sequential set in X . The subsets $A_n, n \in \mathbb{N}$ are called the components of $A(s)$.

Definition 2.7[4] A sequence of fuzzy soft sets is a mapping from \mathbb{N} to the family of all fuzzy soft sets and is denoted by $\{(F, A)_n\}$ or $\{(F, A)_n; n = 1, 2, \dots\}$. That is, $\{(F, A)_n, n \in \mathbb{N}\}$ where $(F, A)_n$ for each $n \in \mathbb{N}$ represents components of fuzzy soft set in $\{(F, A)_n\}$ and $n \in \mathbb{N}$, the set of all natural numbers. A sequence of fuzzy soft sets is called fuzzy soft sequential set.

3. Main Results

Definition 3.1. A fuzzy soft sequential set $\{(F, A)_n\}$ is said to be **fuzzy soft sequential bounded above** if there exists a real number g such that for all $e_i \in A, x_j \in U, \{(F, A)_n\} \lesssim g$. That is, $F_n(e_i)(x_j) \lesssim g$ for all $n \in \mathbb{N}$. Then g is called an **upper bound** of the fuzzy soft sequential set.

Definition 3.2 A fuzzy soft sequential set $\{(F, A)_n\}$ is said to be **fuzzy soft sequential bounded below** if there exists a real number g such that for all $e_i \in A, x_j \in U, \{(F, A)_n\} \gtrsim g$. That is, $F_n(e_i)(x_j) \gtrsim g$ for all $n \in \mathbb{N}$. Then g is called a **lower bound** of the fuzzy soft sequential set.

Definition 3.3 A fuzzy soft sequential set $\{(F, A)_n\}$ is said to be **bounded fuzzy soft sequential set** if it is both fuzzy soft sequential bounded above and fuzzy soft sequential bounded below.

In other words, A fuzzy soft sequential set $\{(F, A)_n\}$ is **bounded fuzzy soft sequential set** iff there exists a real number $g \gtrsim 0$ such that $|\{(F, A)_n\}| \lesssim g$.

Example 3.4. Let us consider the universe $U = \{x_1, x_2\}$ and $A = \{e_1, e_2\} \subset E$ be the set of parameters. Consider $(F, A)_n = \left\{ \left(e_1, \left\{ \frac{x_1}{(1/n)}, \frac{x_2}{(\sqrt{3}/2n)} \right\} \right), \left(e_2, \left\{ \frac{x_1}{(n/n+7)}, \frac{x_2}{(0.75)} \right\} \right) \right\} \forall n \in \mathbb{N}$.

Here $|(F, A)_n| \lesssim 1$ for all $n \in \mathbb{N}$. Hence $\{(F, A)_n\}$ is a bounded fuzzy soft sequential set.

Theorem 3.5. Let (x_n) be a crisp sequence in \mathbb{R} . Let $\{(F, A)_n\}$ be the corresponding fuzzy soft sequential set in \mathbb{R} . Then $\{(F, A)_n\}$ is a bounded fuzzy soft sequential set if and only if (x_n) is bounded.

Proof. Let (x_n) be the given crisp sequence in \mathbb{R} and let $\{(F, A)_n\}$ be the corresponding fuzzy soft sequential set in \mathbb{R} .

Then $\{(F, A)_n\} \in [0, 1]$ if $x_n \in (-\infty, \infty)$.

Assume crisp sequence (x_n) is bounded, then $|(x_n)| \leq k$ for all n . Then, there exists a real number k in \mathbb{R} such that $|\{(F, A)_n\}| \lesssim k$. Hence $\{(F, A)_n\}$ is a bounded fuzzy soft sequential set.

Conversely, assume that $\{(F, A)_n\}$ is a bounded fuzzy soft sequential set. Therefore, there exists a real number k in \mathbb{R} such that $|\{(F, A)_n\}| \lesssim k$. Since $\{(F, A)_n\}$ be the corresponding fuzzy soft sequential set of (x_n) , $|(x_n)| \leq k$ for all n . Thus, (x_n) is bounded.

Definition 3.6. A fuzzy soft sequential set $\{(F, A)_n\}$ is said to be an **increasing** fuzzy soft sequential set if and only if for each natural number n , $(F, A)_n \subset (F, A)_{n+1}$, that is, $(F, A)_1 \subset (F, A)_2 \subset (F, A)_3 \subset (F, A)_4 \subset \dots$

Definition 3.7. A fuzzy soft sequential set $\{(F, A)_n\}$ is said to be **decreasing** fuzzy soft sequential set if and only if for each natural number n , $(F, A)_n \supset (F, A)_{n+1}$, that is, $(F, A)_1 \supset (F, A)_2 \supset (F, A)_3 \supset (F, A)_4 \supset \dots$

Definition 3.8. A fuzzy soft sequential set $\{(F, A)_n\}$ is said to be **monotonic** if and only if the fuzzy soft sequential set is either increasing or decreasing fuzzy soft sequential set.

Example 3.9. Let us consider the universe $U = \{x_1, x_2\}$ and A and $B = \{e_1, e_2\} \subset E$ where $\{e_1, e_2\}$ be a collection of sets of parameters.

Consider $(F, A)_n = \left\{ \left(e_1, \left\{ \frac{x_1}{(1/3n)}, \frac{x_2}{(2/n+1)} \right\} \right), \left(e_2, \left\{ \frac{x_1}{(2/5n)}, \frac{x_2}{(1/n)} \right\} \right) \right\} \forall n \in \mathbb{N}$

implies $(F, A)_1 \supset (F, A)_2 \supset \dots \supset (F, A)_n \supset \dots$. Hence $\{(F, A)_n\}$ is a decreasing fuzzy soft sequential set and also a monotonic.

Consider $(G, B)_n = \left\{ \left(e_1, \left\{ \frac{x_1}{(n/n+4)}, \frac{x_2}{(n/3n+1)} \right\} \right), \left(e_2, \left\{ \frac{x_1}{(0.14)}, \frac{x_2}{(n/n+2)} \right\} \right) \right\} \forall n \in \mathbb{N}$

From the above fuzzy soft sequential set $\{(G, B)_n\}$, we can written as $(G, B)_1 \subset (G, B)_2 \subset \dots \subset (G, B)_n \subset \dots$. Hence $\{(G, B)_n\}$ is an increasing fuzzy soft sequential set and also a monotonic.

Definition 3.10. A fuzzy soft sequential set $\{(F, A)_n\}$ is said to be **eventually contained** in a fuzzy soft set (G, A) if and only if there exists a natural number k such that $(F, A)_n \subset (G, A) \forall n \gtrsim k$.

Example 3.11. Let us consider the universe $U = \{x_1, x_2\}$ and $A = \{e_1, e_2\} \subset E$ be a collection of sets of parameters.

Consider a fuzzy soft set (G, A) , $(G, A) = \left\{ \left(e_1, \left\{ \frac{x_1}{0.2}, \frac{x_2}{0.7} \right\} \right), \left(e_2, \left\{ \frac{x_1}{1}, \frac{x_2}{0.3} \right\} \right) \right\}$

Also consider $(F, A)_n = \left\{ \left(e_1, \left\{ \frac{x_1}{(2/5n)}, \frac{x_2}{(1/3n)} \right\} \right), \left(e_2, \left\{ \frac{x_1}{(n/n+3)}, \frac{x_2}{(1/2n)} \right\} \right) \right\} \forall n \in \mathbb{N}$

Here there exists a natural number 50 such that $(F, A)_n \subseteq (G, A) \forall n \geq 50$.

Definition 3.12 A fuzzy soft sequential set $\{(F, A)_n\}$ in a fuzzy soft topological space (U, τ) converges to a fuzzy soft set (G, A) is said to be **convergence** if it is eventually contained in each neighborhood of the fuzzy soft set (G, A) .

In other words, the fuzzy soft sequential set $\{(F, A)_n\}$ has the limit (G, A) and we write $\lim_{n \rightarrow \infty} (F, A)_n = (G, A)$ or $(F, A)_n \rightarrow (G, A)$ as $n \rightarrow \infty$ or simply $F_n \rightarrow G$ as $n \rightarrow \infty$.

Example 3.13. Let us consider the universe $U = \{x_1, x_2\}$ and $A = \{e_1, e_2\} \subseteq E$ be a collection of sets of parameters and the fuzzy soft topological space $\tau = \{\emptyset, U, (F, A), (G, A)\}$.

Consider the Example 3.11 and define the fuzzy soft set $(F, A) = \left\{ \left(e_1, \left\{ \frac{x_1}{0.3}, \frac{x_2}{0.9} \right\} \right), \left(e_2, \left\{ \frac{x_1}{1}, \frac{x_2}{0.8} \right\} \right) \right\}$.

Here there exists $(F, A) \in \tau$ such that $(G, A) \subseteq (F, A) \subseteq (F, A)$ implies (F, A) is a neighborhood of (G, A) . Also $(F, A)_n \subseteq$ neighborhood of $(G, A) = (F, A) \forall n \geq 50$. Hence $\{(F, A)_n\}$ converges to a fuzzy soft set (G, A) .

Theorem 3.14. If the neighborhood system of each fuzzy soft set in (U, τ) is countable, then the fuzzy soft set (G, A) is open if and only if each fuzzy soft sequential set $\{(F, A)_n\}$ which converges to (F, A) contained in (G, A) is eventually contained in (G, A) .

Proof.

Necessity: Assume $\{(F, A)_n\}$ converges to (F, A) . Since (G, A) is fuzzy soft set in (U, τ) , (G, A) is a neighborhood of (F, A) . Hence $\{(F, A)_n\}$ converges to (F, A) contained in (G, A) is eventually contained in (G, A) .

Sufficiency: For every $(F, A) \subseteq (G, A)$, let $(F, A)_1, (F, A)_2, \dots, (F, A)_n, \dots$ be a neighborhood system of (F, A) . Let $(H, A)_n = \bigcap_{i=1}^n (F, A)_i$. Then $\{(H, A)_n; n = 1, 2, \dots\}$ is a fuzzy soft sequential set which is eventually contained in each neighborhood of (F, A) . Hence there exists a natural number k such that $(H, A)_n \subseteq (G, A) \forall n \geq k$. Thus $(H, A)_n$ are neighborhood's of the fuzzy soft set (F, A) implies (G, A) is a neighborhood of (F, A) . Hence (G, A) is fuzzy soft open set.

Note 3.15. If the neighborhood system of each fuzzy soft set in (U, τ) is **uncountable** in Theorem 3.14, then $(H, A)_n = \bigcap_{n \in \mathbb{N}} (F, A)_n$. Since arbitrary intersection of fuzzy soft open set is not open, which is a contradiction to the neighborhood system of fuzzy soft set.

Definition 3.16. Let $\{(F, A)_n\}$ be a fuzzy soft sequential set over U . Let $n_1 < n_2 < \dots < n_k < \dots$ be a strictly increasing sequence of natural numbers. Define a fuzzy soft sequential set $\{(G, A)_{n_k}\}$ over U is a subsequence of a fuzzy soft sequential set $\{(F, A)_n\}$ such that for all $n \in \mathbb{N}$, $(G, A)_{n_k} = (F, A)_n$ if $n = n_k$ for some $k \in \mathbb{N}$ and equal to zero, otherwise.

That is, for all $n \in \mathbb{N}$, $(G, A)_{n_k} = (F, A)_n$ if $n = n_k$ for some $k \in \mathbb{N}$
 $= \emptyset$, otherwise.

Example 3.17. Let us consider the universe $U = \{x_1, x_2\}$ and $A = \{e_1, e_2\} \subseteq E$ where $\{e_1, e_2\}$ be a collection of sets of parameters.

Consider $(F, A)_n = \left\{ \left(e_1, \left\{ \frac{x_1}{(1/3n)}, \frac{x_2}{(\sqrt{3}/2n)} \right\} \right), \left(e_2, \left\{ \frac{x_1}{(2/5n)}, \frac{x_2}{(1/n)} \right\} \right) \right\} \forall n \in \mathbb{N}$

Let $(G, A)_{n_k} = (F, A)_n$ if $n_k = 2n$ for $k = 2 \forall n \in \mathbb{N}$.

Therefore $\{(G, A)_{n_k}\}$ is a subsequence of a fuzzy soft sequential set $\{(F, A)_n\}$.

Theorem 3.18. The convergence of fuzzy soft sequential set in fuzzy soft topological space (U, τ) satisfies,

- i. Every constant fuzzy soft sequential set $\{(F, A)_n\}$ converges to (F, A) .
- ii. If each subsequence of a fuzzy soft sequential set $\{(F, A)_n\}$ converges to (F, A) , then $\{(F, A)_n\}$ converges to (F, A) .

Proof.

i. Proof is straightforward.

ii. Given each subsequence of a fuzzy soft sequential set $\{(F, A)_n\}$ converges to (F, A)

Claim: $\{(F, A)_n\}$ converges to (F, A)

Assume that $\{(F, A)_n\}$ is not convergent to (F, A) . Then there exists $(G, A) \in \tau$ such that $(F, A) \not\subseteq (G, A)$. Hence there exists a natural number n_0 such that $(F, A)_{n_0} \not\subseteq (G, A) \forall n \geq n_0$. Now we take the set $N_0 = \{n_0 \in \mathbb{N} : (F, A)_n \not\subseteq (G, A)\}$ is infinite and $N_0 = \{n_1 < n_2 < \dots < n_k < \dots\}$. The fuzzy soft sequential set $\{(G, A)_{n_k}\}$ is a subsequence of a fuzzy soft sequential set $\{(F, A)_n\}$ which is not converges to (F, A) , which is a contradiction to our assumption. Hence $\{(F, A)_n\}$ converges to (F, A) .

Definition 3.19. A fuzzy soft sequential set $\{(F, A)_n\}$ is said to be **frequently contained** in a fuzzy soft set (G, A) if and only if for each $n \in \mathbb{N}$, there exists a natural number $k \geq n$ such that $(F, A)_k \subseteq (G, A)$.

Example 3.20. Let us consider the universe $U = \{x_1, x_2\}$ and $A = \{e_1, e_2\} \subseteq E$ be a collection of sets of parameters.

Consider a fuzzy soft set (G, A) , $(G, A) = \left\{ \left(e_1, \left\{ \frac{x_1}{0.5}, \frac{x_2}{1} \right\} \right), \left(e_2, \left\{ \frac{x_1}{0.4}, \frac{x_2}{0.2} \right\} \right) \right\}$

Also consider $(F, A)_n = \left\{ \left(e_1, \left\{ \frac{x_1}{(n/2n+1)}, \frac{x_2}{(n/n+7)} \right\} \right), \left(e_2, \left\{ \frac{x_1}{(0.35)}, \frac{x_2}{(1/7n)} \right\} \right) \right\} \forall n \in \mathbb{N}$

Here for every $n \in \mathbb{N}$ there exists a natural number $k \geq n$ such that $(F, A)_k \subseteq (G, A)$.

Definition 3.21. A fuzzy soft set (G, A) in a fuzzy soft topological space (U, τ) is a **cluster fuzzy soft set** of a fuzzy soft sequential set $\{(F, A)_n\}$ if $\{(F, A)_n\}$ is frequently contained in every neighborhood of the fuzzy soft set (G, A) .

Example 3.22. Let us consider the universe $U = \{x_1, x_2\}$ and $A = \{e_1, e_2\} \subseteq E$ be a collection of sets of parameters and the fuzzy soft topological space $\tilde{\tau} = \{\emptyset, U, (F, A), (G, A)\}$.

Consider the Example 3.20 and define the fuzzy soft set $(F, A) = \left\{ \left(e_1, \left\{ \frac{x_1}{0.7}, \frac{x_2}{1} \right\} \right), \left(e_2, \left\{ \frac{x_1}{0.8}, \frac{x_2}{0.45} \right\} \right) \right\}$.

Here there exists $(F, A) \in \tilde{\tau}$ such that $(G, A) \subseteq (F, A) \subseteq (F, A)$ implies (F, A) is a neighborhood of (G, A) . By the Example 3.20, $\{(F, A)_n\}$ is frequently contained in (G, A) . Since $(G, A) \subseteq (F, A)$, $\{(F, A)_n\}$ is frequently contained in neighborhood of $(G, A) = (F, A)$ implies (G, A) is a cluster fuzzy soft set of a fuzzy soft sequential set $\{(F, A)_n\}$.

Theorem 3.23. If the neighborhood system of each fuzzy soft set in fuzzy soft topological space (U, τ) is countable, then for (G, A) is a cluster fuzzy soft set of a fuzzy soft sequential set, there is a subsequence of fuzzy soft sequential set converging to (G, A) .

Proof. Let $(G, A)_1, (G, A)_2, \dots, (G, A)_n, \dots$ be a neighborhood system of (G, A) . Let $(H, A)_n = \bigcap_{i=1}^n (G, A)_i$. Then $\{(H, A)_n; n = 1, 2, \dots\}$ is a fuzzy soft sequential set such that $(H, A)_{n+1} \subseteq (H, A)_n$ for all $n \in \mathbb{N}$ and is eventually contained in each neighborhood of (G, A) implies $\{(H, A)_n\}$ converges to (G, A) . Since (G, A) is a cluster fuzzy soft set of a fuzzy soft sequential set, $\{(H, A)_n\}$ is frequently contained in each neighborhood of (G, A) . For each $n \in \mathbb{N}$, there exists a natural number k such that $(F, A)_{n_k} \subseteq (H, A)_n$ for $n = n_k$ and hence $\{(F, A)_{n_k}\}$ is a subsequence of a fuzzy soft sequential set $\{(H, A)_n\}$ converges to (G, A) .

Theorem 3.24. Let (G, A) be a cluster fuzzy soft set of a fuzzy soft sequential set $\{(F, A)_n\}$ and (G, A) contained in a fuzzy soft set (F, A) . If (F, A) is fuzzy soft open set, then the fuzzy soft sequential set $\{(F, A)_n\}$ is frequently contained in (F, A) .

Proof. Since (F, A) is open and $(G, A) \subseteq (F, A)$. Hence (F, A) is a neighborhood of (G, A) . Since (G, A) is a cluster fuzzy soft set of a fuzzy soft sequential set $\{(F, A)_n\}$, so by the Definition 3.21, $\{(F, A)_n\}$ is frequently contained in every neighborhood of (G, A) and hence $\{(F, A)_n\}$ is frequently contained in (F, A) .

4. Conclusion

In this paper, we discussed characteristics of sequences in fuzzy soft sequential topological spaces. We imposed Sequences like bounded sequence, increasing sequence, decreasing sequence, convergent sequence, subsequence in fuzzy soft sequential sets. Some properties of the above concepts are also explored.

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