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Research Article

The Varying Volume Content in the Creep Modeling of Thermal and Particle Graded Composite Disk

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ABSTRACT

The purpose of this paper is to give conclusions of effect of the thermal and particle gradient on rotating composite disk made of *AlSiCp* (aluminum silicon carbide particle) by using Sherby's law with varying volume content. The stresses and strains along the radial and tangential components are calculated by using mathematical model. It has been observed that the effect of creep deformation reduces with increasing the volume content in a rotating composite disk and the numerical results are demonstrated graphically.

Keywords: Creep, disk, strain, stress, thermal gradient and particle gradient.

Introduction

FGM developed during a space plane project in 1984 in Japan. These concepts were become popular in Europe and Germany in certain years. Maxel represented the structural units of FGM's material (functionally graded material). There are a large number of engineering applications in FGM. For future power generation industries and high speed spacecraft. FGMs are deemed potential structural material. The first use of metal ceramic FGMs in thermal barrier layer in areas of aerospace and aviation. In FGM constituents vary in some direction thus enabling these material to provide unique performance. FGM have high potential for application in component apply to server thermal and mechanical loading as a result of their unique performance due to spatial tailoring of properties [3]. Functionally Graded Material have various applications in different fields like in automobiles are automobile parts like shock absorber, racing car braces, combustion chamber and spring are made by FGM material. In Energy FGM are act as protecting coating and thermal barrier on turbine blades of gas turbine which is an energy conversion device. FGM are also used in other application like nuclear reactor components, heat exchanges etc.
Rotating disk are used in many instruments for example in mechanical and electronic therefore rotating disks are lot of attention in engineering so rotating disk give a wide region of research because of their immense usage in rotating machinery like ga turbine rotors, flywheels, turbo generator and computer disk dervies [10]. Due to these large number of applications disk operate under high temperature at the same time subjected to elevated stresses obtain by disk rotation [1]. For better material and effective design their is important to study strains and stresses of disks that are under different boundry conditions. The boundary conditions varying with the way of attachment of disk with shaft. These information helps to modify processing parameters of disk material, disk design or change composition of disk material which can improve creep performence of disk. The reasearcher working with rotating disks offer an area of research due to their utiliztion in rotaing machineries [4]. Therefore we are able to reduce creep deformation by varying thickness and properties of disk by using FGM by doing this we can reduce cost and increase the efficiency of disk.

Rattan et al. [12] studies secondary state the creep response of an isotropic FGM rotating disk of *AlSiCP* with using sherby's law for particle gradient. They investigate that FGM disk with parabolic profile have more efficient as compare to constant distribution and linear variations of particle. Khanna et al. [9] investigate secondary state creep in rotating disk having different thickness profile and reinforcement (*SiCp*) gradients. They concluded that with increasing disk thickness gradient radial stress decreases toward internal radii but increases toward circumferential radius, where tangential stress decreases over entire radii. But with increase of $SiCp$ gradient in FGM disk, radial stress but the tangential stress increases towards the internal radii but decreases toward the circumferential radii. The strain rates in disk reduce over the entire radii of disk with increases the thickness gradient or reinforcement gradient of disk. Therefore, they concluded that there is lesser distortion when higher thickness and higher reinforcement gradient in composite disk. Bose et al. [3] investigate the secondary state creep analysis of thermal graded isotropic rotating disk made from linearly varying particle reinforced FGM. They calculate the stress and strain for rotating disk by using von Mises yield criterion and observed that there is small variation for thermal gradient as compare to uniform temperature for radial and tangential stresses. But strain varying significantly in presence of thermal gradient. They conclude that the FGM rotating disk with linearly thermal profile more efficient as compare to uniform distribution. Thakur et al. [14] investigate effect of imposing linear thermal gradient on secondary state creep behavior of rotating FGM disk by using Sherby's law stress and strain rates by using mathematical modeling. They are comparing composite disk having uniform temperature with thermal gradient disk they concluded that the creep rates in rotating FGM disk has been decreases in existence of the thermal gradient.

In this paper, the stable state creep of the rotating disk is supposed for different volume contentat angular speed $\omega = 16000$. Two different cases for changed volume content are discussed and in these two cases three disks are compared. In first case, the disk is operating at constant temperature, in the second disk the disk is operating under thermal gradient 60K = -213.15^{0}C having internal temperature 650K and external temperature 590K and in the third disk is operating under thermal gradient 160K = -13.15^{0}C having internal temperature 650K and external temperature 490K.

1. In case first volume content 10% where $V_n = 20\%$ and $V_x = 10\%$.
2. In case second volume content 15% where $V_n = 30\%$ and $V_n = 15\%$ (i.e. V_n and V_x are particle content at internal and external radius respectively)

2 Mathematical Modeling

2.1 Reinforcement Distribution in the Disk

In current study, an isotropic FGM disk rotating with angular velocity ω having internal radius and circumferential radius r_a = 30mm and r_b = 140mm respectively.The creep constant and density will vary with radial distance.The material properties of disk assumed to function of volume fraction of constituent materials. The composition variation in term of volume percent of SiC, along radial distance V(R), is given as:

$$V(R) = C - DR, r_a \le R \le r_b \tag{1}$$

Where,

$$C = \frac{r_b V_n - r_a V_x}{r_b - r_a} \tag{2}$$

And

$$D = \frac{V_n - V_x}{r_b - r_a} \tag{3}$$

where V_n and V_x are particle content at internal radius and circumferential radius. By mixture of law, density variation in composite is given as,

$$\rho(R) = \rho_p + (\rho_m - \rho_p)\frac{V(R)}{100} \tag{4}$$

$$\rho(R) = \rho_p + (\rho_m - \rho_p)\frac{(C - DR)}{100} \tag{5}$$

where density of matrix alloy is $\rho_m = 3210 kg/m^3$ and density of silicon carbide particles $\rho_a = 2698.9 kg/m^3$. Put the value V(R) from eq^n (1) in eq^n (2) and (3), we get
If average particle content is V_{avg} in FGM disk then,

$$\int_{r_a}^{r_b} 2\pi R V(R) dR = V_{avg}\left(r_b^2 - r_a^2\right) \qquad 6$$

By using the value of V(R) from eq^n (1) into eq^n (6), we get following relation:

$$V_{avg} = C - \frac{2}{3}\frac{D\left(r_b^3 - r_a^3\right)}{r_b^2 - r_a^2} \qquad 7$$

2.2Creep Law

The stable state creep response of the AlSiCp composite of varying composition have been described in the given form terms of Sherby's law [13],

$$\dot{\varepsilon} = \left[M_c\left(\bar{\sigma} - \sigma_0\right)\right]^8 \qquad 8$$

where.effective stress is σ, threshold stress is σ_0 and effective strain rate is ε.

2.3Creep Specification

Creep specification is given by:

$$M_c = \frac{1}{E}\left[\frac{A D_L \lambda^3}{\left|\vec{b}_r^5\right|}\right]^{\frac{1}{8}} \qquad 9$$

where M_c material creep constant, A is constant,young's modulus is E, $\left|\vec{b}_r\right|$ is the magnitude of Burger's vector , lattice diffusivity is D_L. The values of creep specification M_c and σ_0 has been obtained from the creep results reported for $AlSiC_p$ composite [11] , these has been fitted by the following regression eq^n which is function of particle size is $P = 1.7\mu m$, temperature is T and volume is V [8].

$$In\left(M_c(R)\right) = -34.91 + 0.2112 InP + 4.89 InT(R) - 0.591 InV(R) \qquad 10$$

$$\sigma_0(R) = -0.02050P + 0.0378T(R) + 1.033V(R) - 4.9695 \qquad 11$$

2.4Thermal Gradient

Condition for thermal gradient disk is given by

$$T(R) = F - GR, r_a \le R \le r_b \qquad 12$$

Where,

$$F = \frac{r_b T_n - r_a T_x}{r_b - r_a} \qquad 13$$

And

$$G = \frac{T_n - T_x}{r_b - r_a} \qquad 14$$

where T_n is temperature at internal radii and T_x is temperature at the circumferential radii.

3.Mathematical Formulation

Consider a disk of AlSiCp of constant width having internal radii r_a , circumferential radii r_b revolving at angular speed ω ,
Given following hypothesis are made for the modeling: Assumed that the stress is at steady state.
Elastic deformation is small can be neglected as compared to creep deformation for the disk. Biaxial state at any point of the disk of stress exists.
The disk show a steady state creep behavior, which can be described by Sherby's constitutive model as given by equation (8)
The generalized constitutive equations for an isotropic rotating disk are:

$$\dot{\varepsilon}_R(R) = \frac{\dot{\varepsilon}}{2\bar{\sigma}}\left[2\sigma_R(R) - \left(\sigma_\theta(R) + \sigma_z(R)\right)\right] \qquad 15$$

$$\dot{\varepsilon}_\theta(R) = \frac{\dot{\varepsilon}}{2\bar{\sigma}}\left[2\sigma_\theta(R) - \left(\sigma_z(R) + \sigma_R(R)\right)\right] \qquad 16$$

$$\dot{\varepsilon}_z(R) = \frac{\dot{\varepsilon}}{2\bar{\sigma}}\left[2\sigma_z(R) - \left(\sigma_R(R) + \sigma_\theta(R)\right)\right] \qquad 17$$

where strain rates are ε_R , ε_θ and ε_z [5] and stress rates σ_R, σ_θ and σ_z corresponding in direction R, θ, z which indicate by the subscripts.For bixial state of stress $(\sigma_z = 0)$ .The effective stress $\bar{\sigma}$ is based on Mises criterion (1913) are given below for the biaxial state of stress,

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\left[\sigma_R^2(R) + \sigma_\theta^2(R) + \left(\sigma_R(R) - \sigma_\theta(R)\right)^2\right]^{\frac{1}{2}} \qquad 18$$

Using equations (17) and (9) in Equation (15) and (16), we get,

$$\dot{\varepsilon}_R(R) = \frac{\left[M_c(R)\left(\bar{\sigma} - \sigma_0(R)\right)\right]^8 \left(2w(R) - 1\right)}{2\left[w^2(R) - w(R) + 1\right]^{\frac{1}{2}}} \qquad 19$$

$$\dot{\varepsilon}_\theta(R) = \frac{\left[M_c(R)\left(\bar{\sigma} - \sigma_0(R)\right)\right]^8 \left(2 - w(R)\right)}{2\left[w^2(R) - w(R) + 1\right]^{\frac{1}{2}}} \qquad 20$$

$$\dot{\varepsilon}_z(R) = -\left(\dot{\varepsilon}_R(R) + \dot{\varepsilon}_\theta(R)\right) \qquad 21$$

Axial strain rate in equation (21) is just defined according to the constitutive equation, otherwise it is assumed zero in the present research work. Where, at any radius R ratio of radial stress and tangential stress is

$$w(R) = \frac{\sigma_R(R)}{\sigma_\theta(R)} \qquad 22$$

The stability of forces in radial direction of element implies that

$$\frac{d}{dR}\left[R\sigma_R(R)\right] - \sigma_\theta(R) + \rho(R)\omega^2 R^2 = 0 \qquad 23$$

$$\sigma_\theta(R) = \frac{\dot{\mu}_a^{\frac{1}{8}}}{M_c(R)}\psi_1(R) + \psi_2(R) \qquad 24$$

$$\dot{\mu}_a^{\frac{1}{8}} = \frac{\int_{r_a}^{r_b} M(R)\sigma_\theta(R)dR - \int_{r_a}^{r_b} M(R)\psi(R)dR}{\int_{r_a}^{r_b} \psi_1(R)dR} \qquad 25$$

$$\psi_1(R) = \frac{\psi(R)}{\left[w^2(R) - w(R) + 1\right]^{\frac{1}{2}}} \qquad 26$$

$$\psi_2(R) = \frac{\sigma_0(R)}{\left[w^2(R) - w(R) + 1\right]^{\frac{1}{2}}} \qquad 27$$

$$\psi(R) = \left[\frac{2\left(w^2(R) - w(R) + 1\right)^{\frac{1}{2}}}{R\left(2 - w(R)\right)} \exp\left(\int_{r_a}^{R} \frac{g(R)}{R} dR\right)\right]^{\frac{1}{8}} \qquad 28$$

And

$$g(R) = \frac{2w(R) - 1}{2 - w(R)} \qquad 29$$

After finding value of $\sigma_\theta(R)$ tangential stress and value of radial stress $\sigma_R(R)$ can be calculated from equation (23)

$$\sigma_R(R) = \frac{1}{R}\int_{r_a}^{R} \sigma_\theta dR - \frac{\omega^2}{R}\left[\frac{R^3 - r_a^3}{3}\left(\rho_p + \left(\rho_m - \rho_p \frac{C}{100}\right)\right) - \frac{\rho_m - \rho_p}{100}\frac{D\left(R^4 - r_a^4\right)}{4}\right] \qquad 30$$

4.Discussion and graphical representation

The effect of imposing thermal gradient as well as particle gradient on rotating composite disk are shown in figures given below

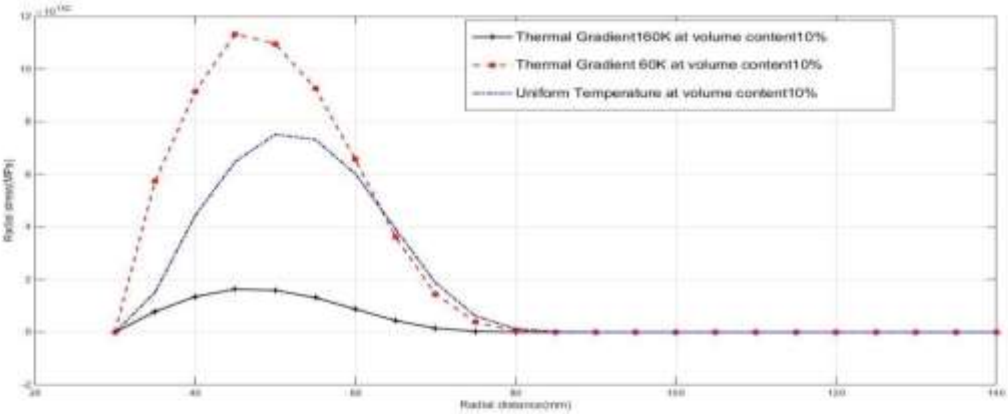

Figure 1: The variation of the radial stress with the radial distance for disk at uniform temperature and thermal gradient with volume content 10%

The graph drawn between radial stresses and radial distance at constant temperature, thermal gradient 60K and 160K having volume content 10% the figure 1, shows that the radial stress is more at center as compare to internal and external radii.

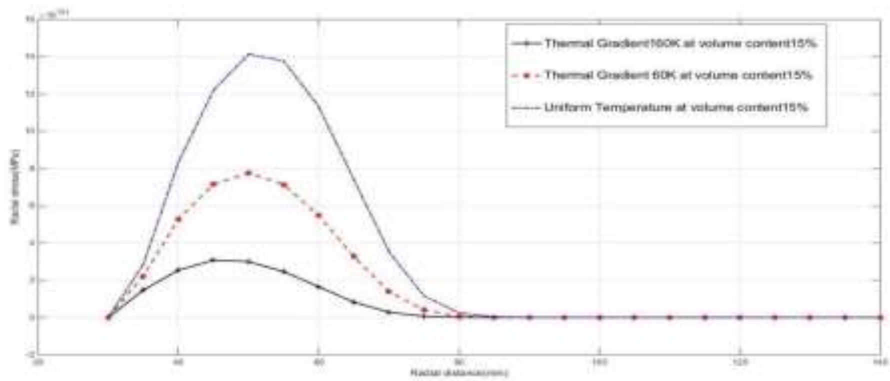

Figure 2: The variationsof radial stress with radial distance for disk at uniform temperature and thermal gradient with volume content 15%

The graph drawn between the radial stresses and radial distance at constant temperature, thermal gradient 60K and 160K having volume content 15% the figure 2, shows that the radial stress is more at center as compare to internal and external radii.

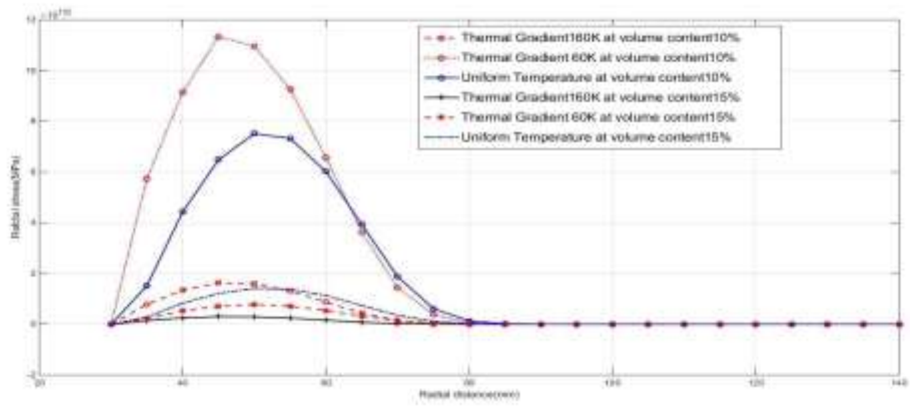

Figure 3: The variations of radial stresses with radial distance for disk at uniform temperature and thermal gradient with volume content 10% and 15%

The graph drawn between the radial stresses and radial distance at constant temperature, thermal gradient 60K and 160K having volume content 10% and 15% the figure 3, shows that the radial stress is more at center as compare to internal and external radii.It has been concluded that with the increases in volume content the radial stresses start decreases.

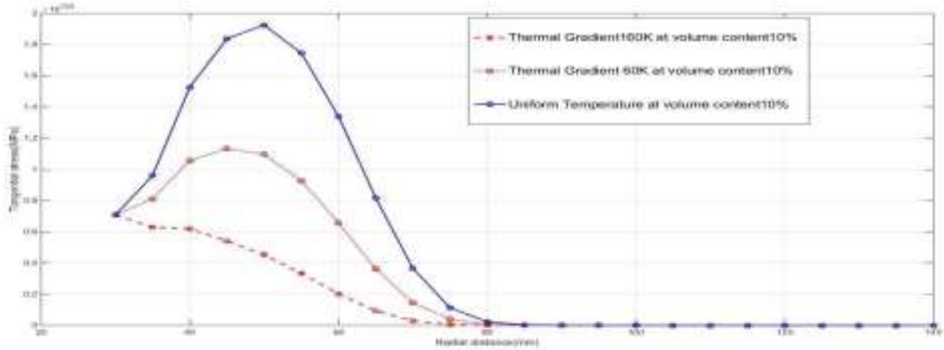

Figure 4: The variations of the tangential stresses with radial distance for disk at uniform temperature and thermal gradient with volume content 10%

The graph drawn between the tangential stresses and radial distance at constant temperature, thermal gradient 60K and 160K having volume content 10% the figure 4, shows that the tangential stress is decreases as we move from internal to external radii.

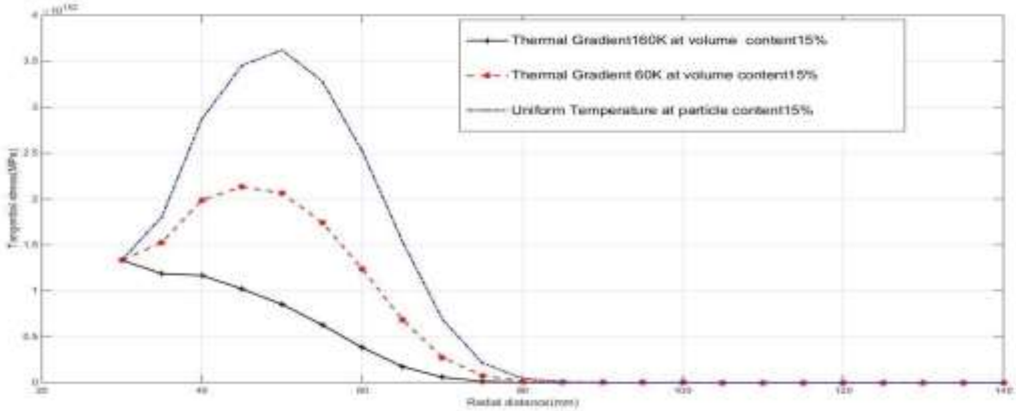

Figure 5: The variations of the tangential stresses with radial distance for disk at uniform temperature and thermal gradient with volume content 15%

The graph drawn between the tangential stresses and radial distance at constant temperature, thermal gradient 60K and 160K having volume content 15% the figure 5, shows that the tangential stress is decreases as we move from internal to external radii.

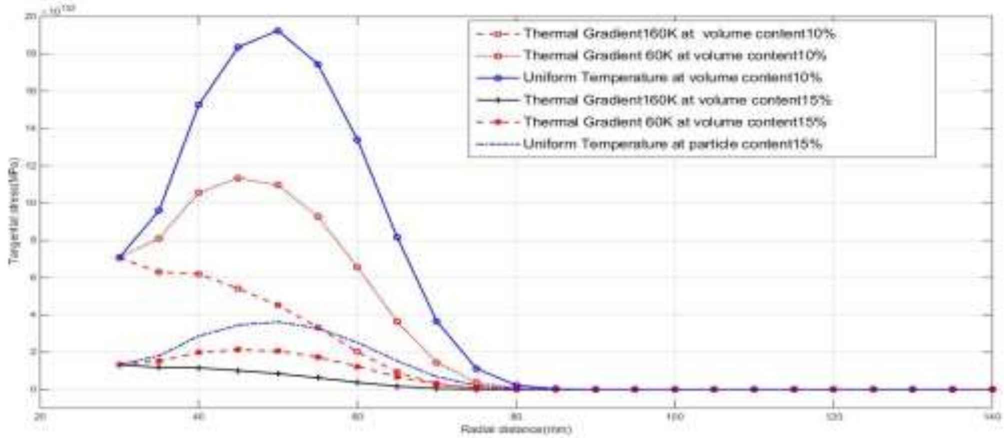

Figure 6: The variations of the tangential stresses with radial distance for disk at uniform temperature and thermal gradient with volume content 10% and15%

The graph drawn between the tangential stresses and radial distance at constant temperature, thermal gradient 60K and 160K having volume content 10% and 15% the figure 6, show that tangential stress is decreases with increases volume content.

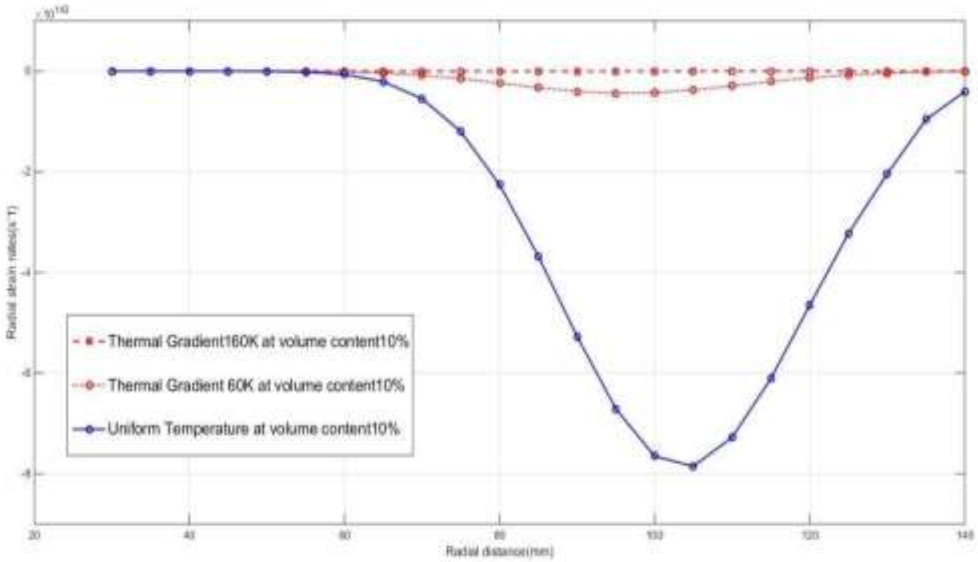

Figure 7: The variations of the radial strain rates with radial distance for disk at uniform temperature and thermal gradient with volume content 10%.

The graph drawn between the radial strain rates and radial distance at constant temperature, thermal gradient 60K and 160K having volume content 10% the figure 7, shows that the radial strain rates decreases at center of composite disk as we move from internal to external radii.

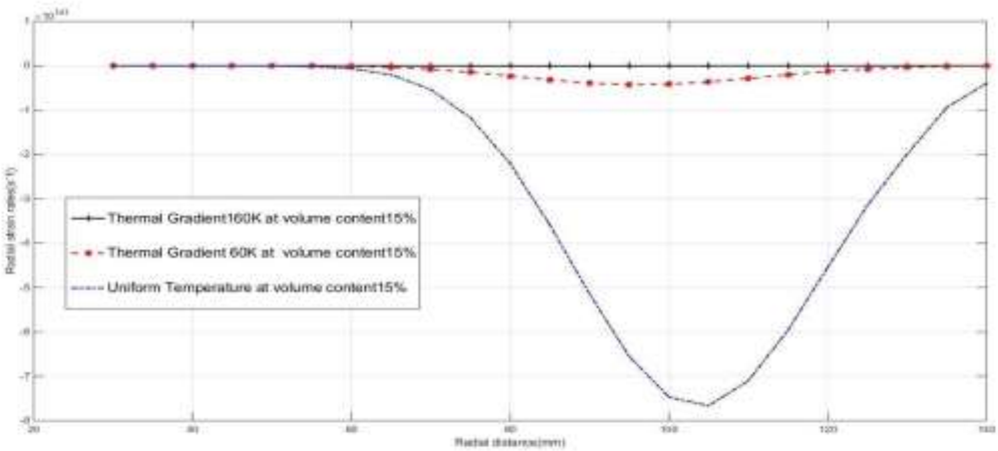

Figure 8: The variations of the radial strain rates with radial distance for disk at uniform temperature and thermal gradient with volume content 15%.

The graph drawn between the radial strain rates and radial distance at constant temperature, thermal gradient 60K and 160K having volume content 15% the figure 8, shows that the radial strain rates decreases at centre of composite disk as we move from internal to external radii.

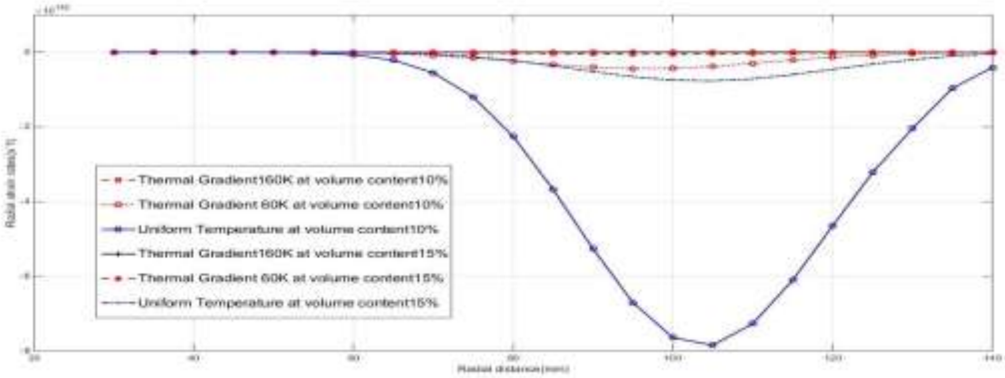

Figure 9: The variations of the radial strain rates with radial distance for disk at uniform temperature and thermal gradient with volume content 10% and 15%.

The graph drawn between the radial strain rates and radial distance at constant temperature, thermal gradient 60K and 160K having volume content 10% and 15% the figure 9, shows that radial strain rates increase with increases volume content.

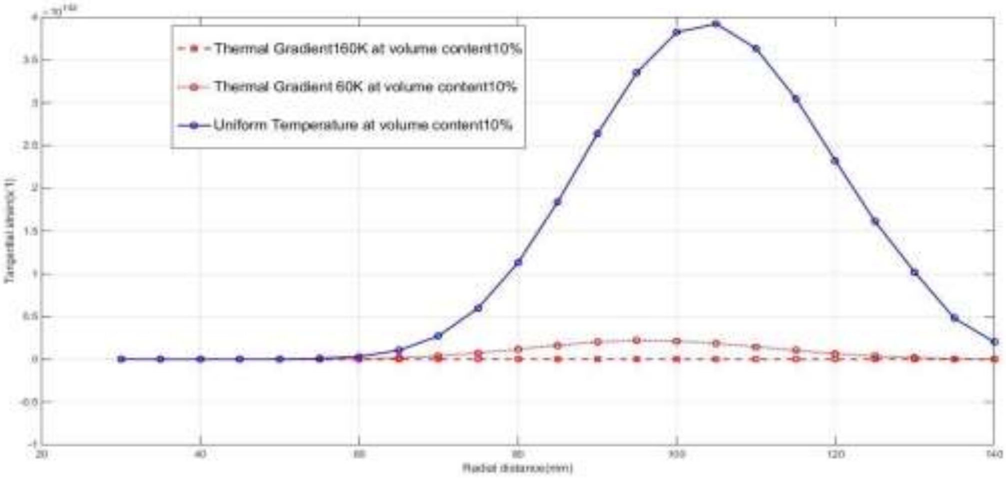

Figure 10: The variations of the tangential strain rates with the radial distance for disk at uniform temperature and thermal gradient with volume content 10%.

The graph drawn between the tangential strain rates and radial distance at constant temperature, thermal gradient 60K and 160K having volume content 10% the figure 10, shows that tangential strain rates increases at the centre of composite disk as we move from internal to external radii.

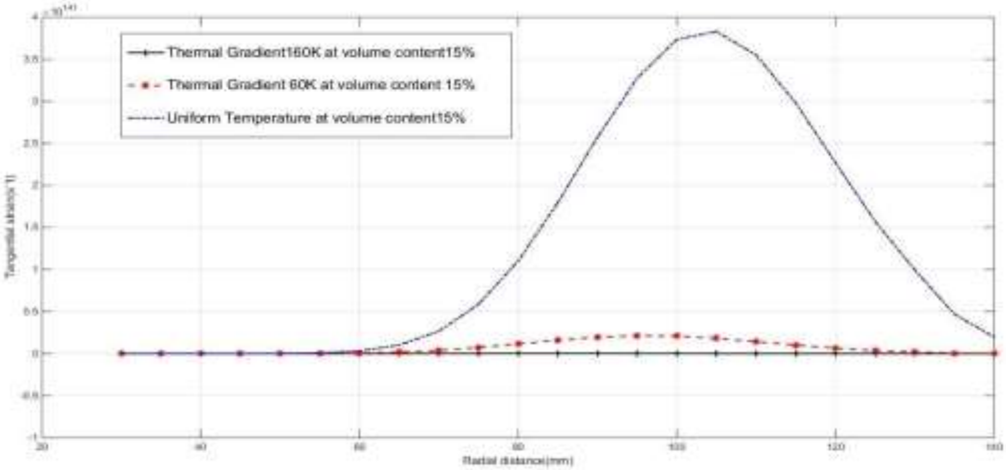

Figure 11: The variations of the tangential strain rates with the radial distance for disk at uniform temperature and thermal gradient with volume content 15%.

The graph drawn between the tangential strain rates and radial distance at constant temperature, thermal gradient 60K and 160K having volume content 15% the figure 11, shows that tangential strain rates increases at the centre of composite disk as we move from internal to external radii.

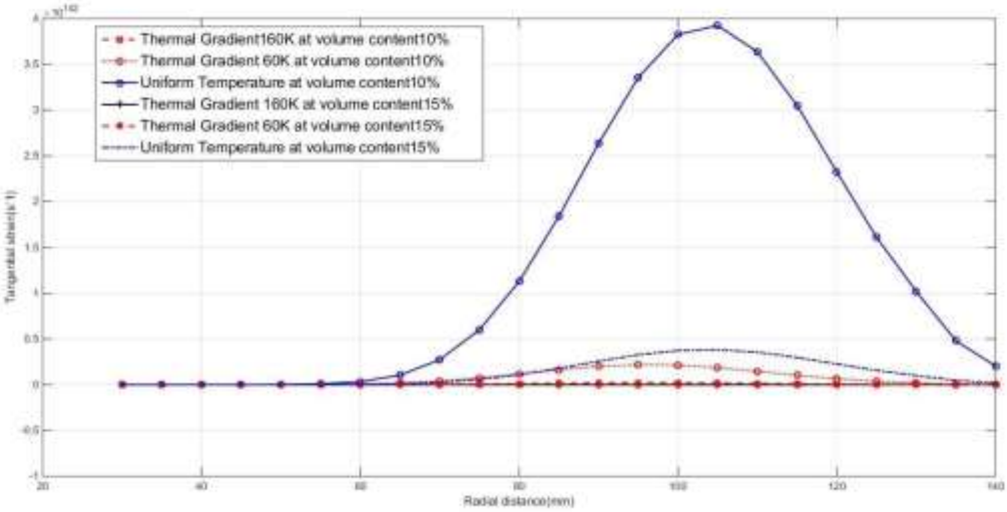

Figure 12: The variations of the tangential strain rates with the radial distance for disk at uniform temperature and thermal gradient with volume content 10% and 15%.

The graph drawn between the tangential strain rates and radial distance at constant temperature, thermal gradient 60K and 160K having volume content 10% and 15% the figure 12, shows that that tangential strain rates decreases with increasing volume content.

1. Conclusion

It has been observed that with increase in volume content for thermally graded *Al–SiCp*disk the radial, tangential stresses and strain rates will ease. This outcome helps to reduce distortion in the composite disk.

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